Numerical modeling of double-mode RR Lyrae stars

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Abstract. We used the Florida nonlinear hydrodynamical code with turbulent convection to model RR Lyrae stars. This approach naturally gives double-mode (DM) solutions. We investigated series of models with different parameters (metallicity, luminosity, etc.). Our goal was to estimate the width of the double-mode region. The resulted tendencies can be compared directly with observational properties.

Keywords: RR Lyrae, Double-mode pulsations, Turbulence, Convection, numerical modeling, pulsating stars, variable stars

1. Introduction

Modeling of double-mode (DM) radial pulsation of classical variable stars was a long-standing enigma of stellar astrophysics. In the late eighties and early nineties simple nonresonant **amplitude equations** turned out to be able to describe the double-mode pulsations (Buchler & Kovács 1986). In spite of great efforts, purely radiative hydrocodes failed to capture the phenomenon. Including 1-D, time dependent description of **turbulent convection** in the models recently led to a breakthrough in pulsation theory by naturally providing DM oscillations. (DM Cepheids: Kolláth et al. 1998, RRd stars: Feuchtinger 1998).

2. Nonlinear survey of double-mode RR Lyrae

Numerical recipes treating turbulent convection contain several numerical parameters (e.g. α_ν is related to eddy viscosity). The physics of the convection could be revealed by fine-tuning these parameters in the models to fit the observational constrains. From the observational point of view various galactic and extragalactic cluster-variable populations representing different metal-content have recently become available thanks to large-scale photometric surveys. Therefore it's useful to investigate the influence of both observable and numerical parameters

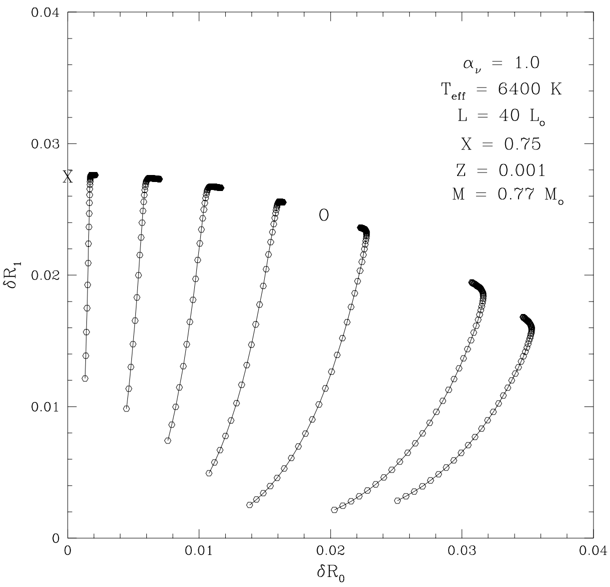

Figure 1. Evolution of the modal amplitudes for different initial conditions. **O** denotes the stable double-mode fixpoint, **X** shows the unstable first overtone mode fixpoint.

to the theoretical DM RR Lyrae region (hereafter DMR) of the instability strip. **The aim of this work was to map the topology of the RRd area in the** *effective temperature - luminosity - metallicity - eddy viscosity* **parameter-space at a given stellar mass.**

We used the implicit, 1-D Lagrangian, nonlinear hydrodynamical code described in detail in Buchler et al. (1997), including turbulent convection (Yecko, Kolláth & Buchler 1998) to model RRd stars. First the linear models were built up, then the hydrodynamical behavior of these models were integrated through 750-1500 periods applying different combinations of fundamental mode (FM) and first overtone mode (OT) eigenvectors as initial perturbations. After that the width of the double-mode instability strip was determined examining the fixpoints on the $\delta R_0(t) - \delta R_1(t)$ planes, where δR_i denotes the amplitude of the relative variation of stellar radius due to the FM and the OT pulsation, respectively (Figure 1.).

3. Results

We computed sequences of models covering a reasonable range of observable and numerical parameters (luminosity: L= 35-55 $L_\odot$, metallic-

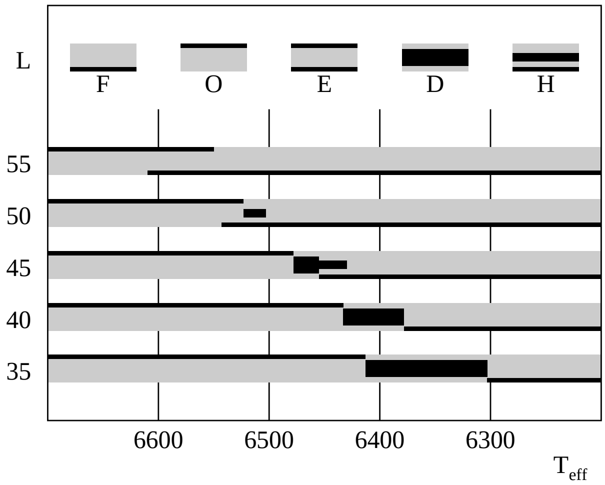

Figure 2. DM instability region at Z=0.0001, α_ν=1.0. F: fundamental mode, O: first overtone, E: either fundamental mode or first overtone, D: double-mode, H: either DM or fundamental mode

ity: $Z = 10^{-4} - 4 \times 10^{-3}$, and turbulent eddy viscosity: $\alpha_\nu = 0.2 - 1.8$). Horizontal branch stellar masses were fixed at $M = 0.77 M_\odot$. Hydrogen-content was also constant during this work (X=0.75). The calibration of the luminosity-metallicity relation is based on the empirical relation of Jurcsik (1998a) and the paper of Dorman (1992) was used for the [Fe/H] - Z conversion for horizontal-branch stars. The following numerical (dimensionless) parameters were used: number of zones: 120, $\alpha_l = 0.41$, $\alpha_c = 0.75$, $\alpha_s = 0.75$, $\alpha_t = 1.0$, $\alpha_d = 4.0$, $\alpha_p = \frac{2}{3}$. The following **tendencies** are apparent from our sequences:

- An increase in **luminosity** yields narrower DMR and shifts it to higher temperatures (Figure 2.). After the vanishing of DM pulsation a broad *either-or-region* appears.

- Surprisingly the effect of **metallicity**-parameter is rather small in the $Z = 10^{-4} - 4 \times 10^{-3}$ range.

- The shape of the α_ν - T_{eff} diagram is similar to Figure 2.: an increase in **eddy viscosity** pushes the DMR to higher T_{eff}. Another interesting feature is again the apparent either-or-region at the blue edge of the DMR at low α_ν.

- In order to compare the **location of the DMR** with observational results, models of lower masses were investigated and similar surveys of lower masses (Bono et al. 1995 and Feuchtinger 1998) were taken into account. This comparison revealed that the DMR at

0.65 $M_\odot$ is shifted to the blue by 150-200 K. In this way excellent agreement can be seen with the position of observed RRd stars in M3 (Bakos & Jurcsik 1999).

– Nevertheless the empirical and theoretical slopes and orientations of the **blue edge of the fundamental IS** are still discrepant at the $L = 35 - 55 L_\odot$ interval (Jurcsik, 1998b).

4. Conclusions

Luminosity primarily **controls** the volume of the region of possible DM-pulsation in parameter-space at fixed stellar mass. The role of **eddy viscosity** is similar to that of luminosity at a given stellar mass. On the other hand **metallicity** has much less effect on the extent of DMR. The outcome of this survey (DMR location and width) is in fairly **good agreement with former observational and other theoretical results** in many respects. As disturbing discrepancies still remained, it seems that fundamental improvements should be done in the code to achieve further convergence between theory and observations.

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