An alternative mathematical treatment of the modulated RR Lyrae stars

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Abstract

We present a different analytical description of the Blazhko RR Lyrae stars's light curves in which we take into account both the amplitude and phase modulation directly.
Main advantages:

- **To explain global light curve behaviours such as**
  - **average brightness variation,**
  - **non-linear shape of the envelope curve,**
  - **non-sinusoidal frequency variation (see Szabó et al. 2009).**
- **Explanation for Fourier properties such as**
  - **appearing higher order side frequencies,**
  - **uneven amplitudes of side peaks,**
  - **larger amplitudes of higher order peaks than the lower order ones,**
  - **appearing modulation frequency itself and its harmonics.**
- **Reduce the number of necessary parameters by a factor of 10.**

Conventional description

The light curve of an RR Lyrae star is conventionally described by a Fourier series of limited number of terms. In the case of modulated (Blazhko) RR Lyrae stars the Fourier sum includes terms of harmonics of the main pulsation frequency, side frequencies due to the modulation and modulation frequency itself:

$$m(t) = \frac{a_0}{2} + \sum_{i=1}^{n} a_i \sin(2\pi f_i t + \varphi_i), \qquad (1)$$

where $f_i = kf_0 + lf_m$, $k = 0, 1, 2, \dots$, $l = 0, \pm1, \pm2, \dots$; f_0 and f_m are the main pulsation frequency and modulation one, respectively. The number of necessary parameters (amplitudes and phases) could reach as many as 500-600 for a long time series of good quality. A good example is the CoRoT data of V1127 Aql, an RRab star showing strong modulation both in its amplitude and phase (Chadid et al. 2009).

Coursebook modulations

Modulation is a technique used in electronic communication, mostly for transmitting information signal via a radio carrier wave (see e.g. Newkirk & Karlquist 2004).

- Amplitude modulation (AM) is the simplest of the three known cases. The transmitter uses the information signal $U_m(t)$ to vary the amplitude of the carrier U_c to produce a modulated signal:

$$U_{AM}(t) = [U_c + U_m(t)] \sin(2\pi f_c t + \varphi_c).$$

Fourier spectrum of an AM signal is an equidistant triplet, where the amplitude of the side-bands are always equal to each other and smaller than the carrier's one. The amplitude of the carrier wave is constant.

- Frequency modulation (FM) uses the information signal $U_m(t)$ to vary the carrier frequency within some small range about its original value.

$$U_{FM}(t) = U_c \sin[2\pi (f_c + \eta U_m(t)) t + \varphi_c],$$

where modulation index is defined as $\eta = \Delta f / f_m$. Frequency modulation (FM) and phase modulation (PM) are equivalent if the modulation frequency is fixed in time.
Fourier spectrum of an FM signal is highly depend on the value of modulation index. If $\eta < 0.5$, spectrum is an equidistant triplet, but side peaks could have different amplitudes. If $\eta > 0.5$, higher order side peaks at $f_c \pm 2f_m$, $f_c \pm 3f_m \dots$ appear. When η is increasing the amplitude of side peaks is also increasing, but the amplitude of the carrier decreasing. On the one hand the side peaks could be larger than the carrier's one, on the other hand higher order side peaks could also be larger amplitude than the lower order ones.

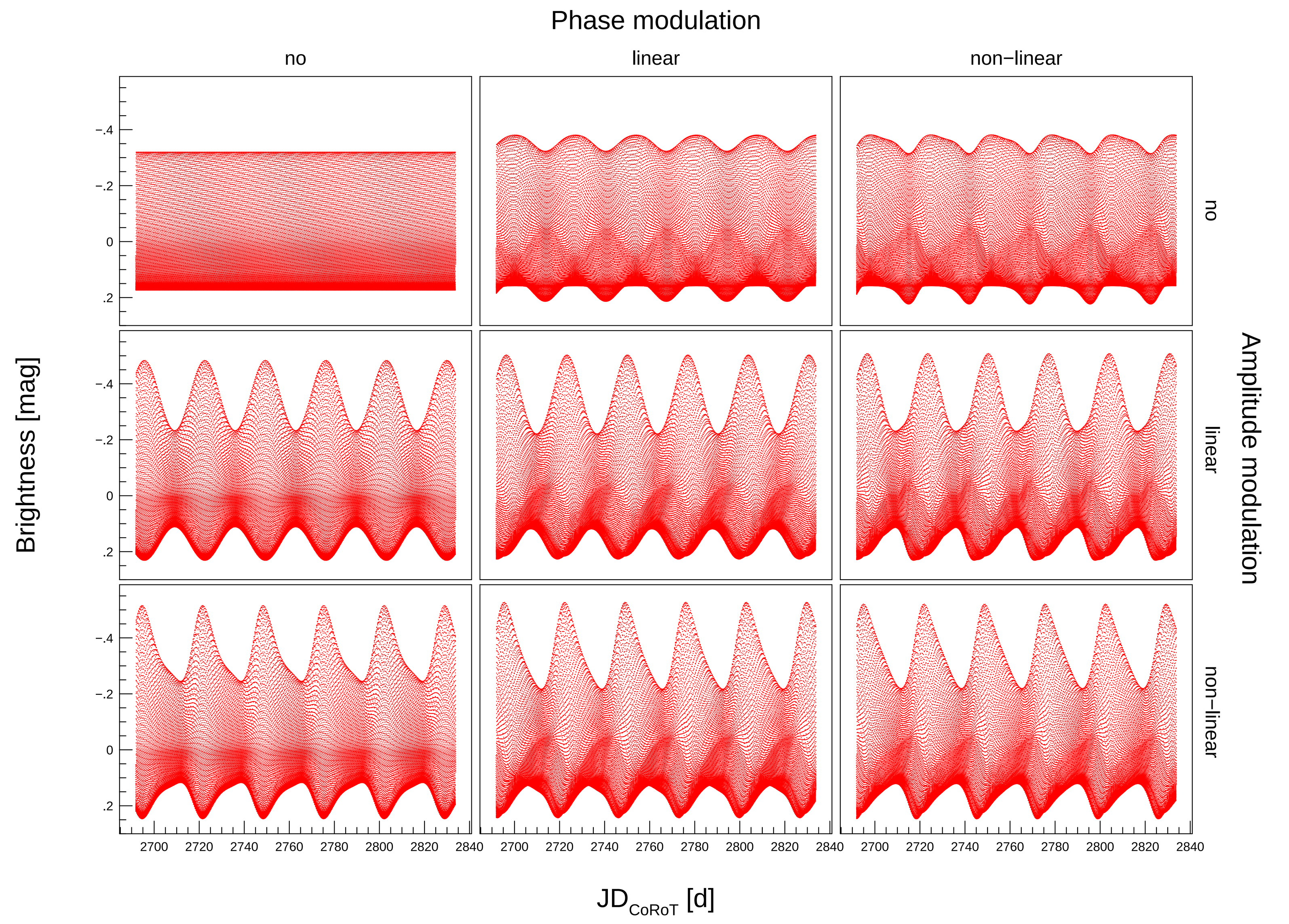

Figure 1: *Artificial light curves calculated from Eq. (2) on the time series of the observing run of the CoRoT LRc01. From left to right the phase modulation, from top to bottom the amplitude modulation are changed as non-modulated, linear and non-linear, respectively.*

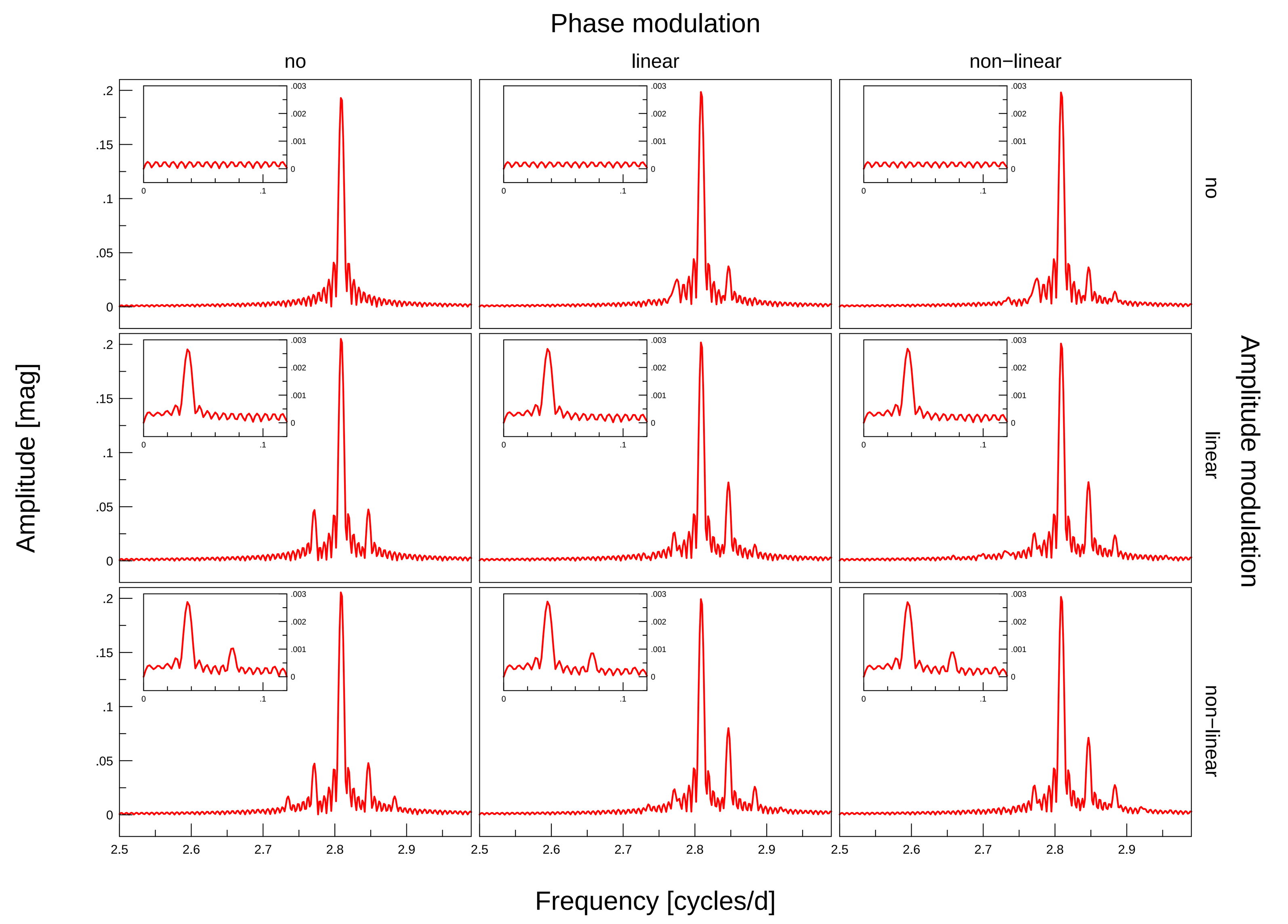

Figure 2: *Fourier amplitude spectra of the above light curves around the main pulsation frequency f_0 (normal panels) and modulation frequency f_m (inserts).*

Blazhko modulation

As it is well known, in the case of Blazhko RR Lyrae stars either AM and FM/PM could be present. We applied the above sketched frame for describing the modulations. The pure pulsation signal with the frequency of f_0 and its harmonics was used as a "carrier wave". We assumed that both the amplitude and phase modulation have the same and constant frequency f_m. Although there are some stars, where f_m changes in time (LaCluyzé at al. 2004, Jurcsik et al. 2002 and references therein), the long time scale of the effect allowed us (as a first approximation) to neglect it. To combine the modulation formulae with these assumptions we obtained:

$$m(t) = \left[\frac{a_0^A}{2} + \sum_{j=1}^{m} a_j^A \sin\left(2\pi j f_m t + \varphi_j^A\right)\right] \left[\frac{a_0}{2} + \sum_{i=1}^{n} a_i \sin\left(2\pi i f_0 t + \varphi_i + \frac{1}{i}\sum_{k=1}^{l} a_k^P \sin\left(2\pi k f_m t + \varphi_k^P\right)\right)\right], \quad (2)$$

where $i, j, k = 1, 2, \dots$ are independent indices.

We generated artificial light curves to test expression (2). Some typical results are shown in Fig. 1. Here we used the parameters f_0, a_i, φ_i of the CoRoT light curve of V1127 Aql for a carrier (left upper panel in Fig. 1). Starting from here the panels in the columns use one (linear) or more (non-linear) terms from the phase modulation in Eq. (2). Amplitude modulation in the same way are given in the panels of lines. Characteristic details of the Fourier spectra of the previous cases in Fig. 1 panels are shown in Fig. 2. Special cases (even – uneven amplitudes, triplet – multiplet fine structure, etc.) of the most general possibilities (summarized in the Abstract) are presented in Fig. 1 and Fig. 2.

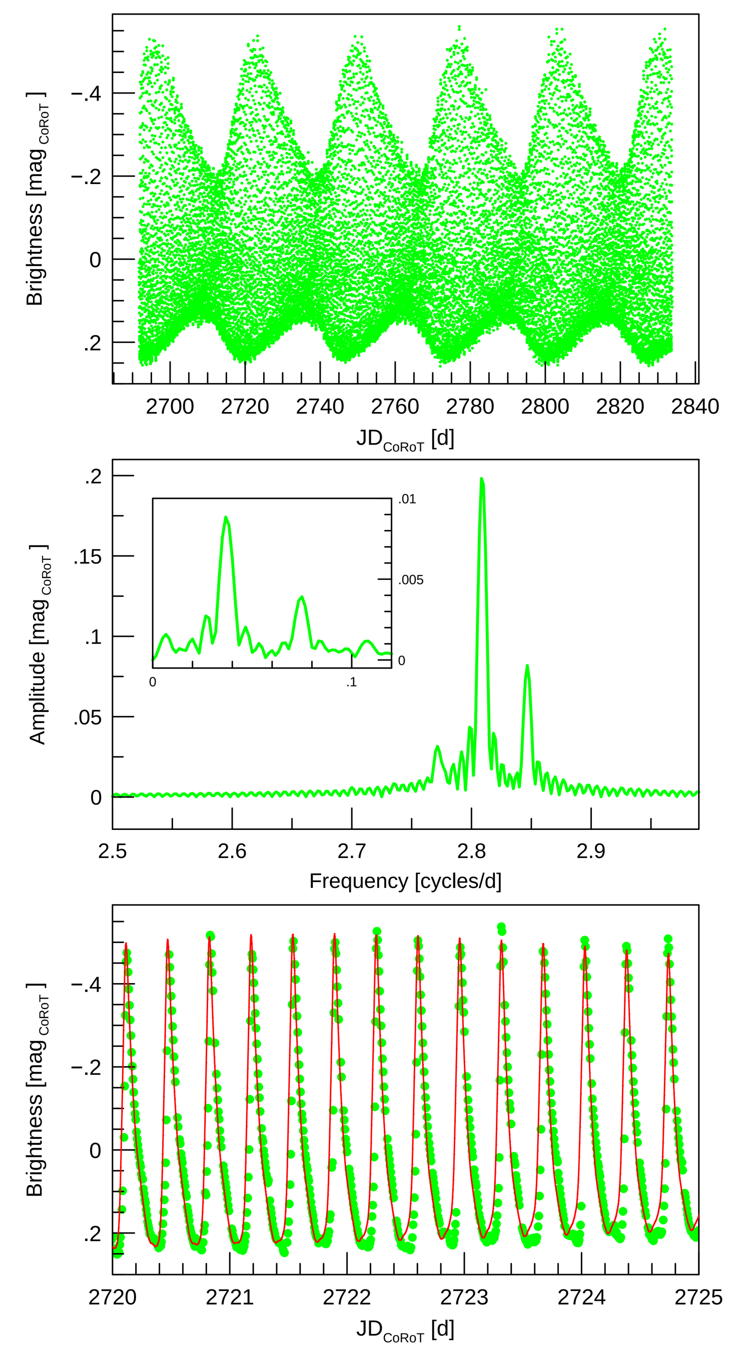

Figure 3: *(top) The observed CoRoT light curve of V1127 Aql. (middle) The Fourier amplitude spectrum of the data around its main pulsation frequency $f_0 = 2.8090169$ d^{-1} and modulation $f_m = 0.037232$ d^{-1} (insert). (bottom) A part of the fit the observed (green) and modelled by Eq. 2. (red) light curves.*

Application to V1127 Aql

We model the light curve behaviour of the CoRoT observations of the star V1127 Aql as a test. We focused on the Blazhko phenomenon only and finer effects were neglected. To find the model parameters we need to solve the non-linear least-squares problem using Eq. (2) and the observed data. It was technically realized by the Levenberg-Marquardt algorithm (Press et al., 1992). In Fig. 3 the observed light curve of V1127 Aql (top), its Fourier spectrum around the main pulsational (middle) and modulation (insert) frequencies and the fit of the light curve (bottom) is presented. Since the harmonics of f_m are found in the Fourier spectrum and also the frequency variation is non-linear (see Szabó et al. 2009) we have to use the most general case (right bottom panel) of Fig. 1&2 to fit the pulsation behaviour of V1127 Aql, namely non-linear amplitude and phase modulations.

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