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Multicolour CCD photometry of the variable stars in globular cluster M3

J. M. Benkő[1]⋆, G. Á. Bakos,[1,2,3] and J. Nuspl[1]
[1]*Konkoly Observatory of the Hungarian Academy of Sciences, P O Box 67, H-1525 Budapest, Hungary*
[2]*Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA02138, USA*
[3]*Hubble Fellow*

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ABSTRACT
We present time series data on the variable stars of the galactic globular cluster Messier 3 (M3). We give $BVI_{\rm C}$ light curves for 226 RR Lyrae, 2 SX Phe and 1 W Vir type variables, along with estimated fundamental photometric parameters such as intensity and magnitude-averaged brightness and pulsation periods. In some cases the periods we have found significantly differ from the previously published ones. This is the first published light curve and period determination for variable V266. The *I*-band light curve has not been observed previously for numerous (76) variables. Three new RR Lyrae variables have been discovered. Groups of RR Lyrae variables that belong to different evolutionary stages and have been separated previously on the basis of *V* data were found here for all colours and colour indices by cluster analysis. The *I*-band period – luminosity relation is also discussed. From the 66 modulated (Blazhko type) RR Lyrae stars we investigated, six are newly identified and two of them are first overtone pulsators. In the case of 13 RR Lyrae, the period of Blazhko cycle has been estimated for the first time. V252 is identified as a new RRd variable. Amplitude ratio of RRd stars have been investigated to search possible mode content changes. In contrast to previous publications no changes have been found. Problems with the sampling of the time series of typical cluster variability surveys is demonstrated.

Key words: techniques: photometric – stars: RR Lyr – stars: variables – globular clusters: individual: M3

1 INTRODUCTION

Multicolour photometry of RR Lyrae stars is very important for obtaining information on their physical parameters such as temperature, mass and luminosity. RR Lyrae stars in globular clusters deserve additional interest because they offer an excellent test for the stellar evolution theory and cosmological distance scale. Regular monitoring of clusters' variables is also essential to investigate various subtle effects, such as period and modal content changes of multimode RR Lyrae stars, or the Blazhko phenomenon.

M3 (NGC 5272; $\alpha_{2000} = 13^{\rm h}42^{\rm m}11\overset{\rm s}{.}2$, $\delta_{2000} = 28^\circ22'32''$) has been target of a large number of different studies. Although it is the most variable-rich galactic globular cluster (with 274 catalogued variables, with most of these belonging to the RR Lyrae type), the number of high quality time series observations are still limited. Corwin & Carney (2001) (hereafter CC01) published a comparable amount of data in *B* and *V* colours to the dataset presented here. In addition, only three CCD-observations based publications exist with partial coverage of M3: Kaluzny et al. (1998) (K98) in *V*-band, Carretta et al. (1998) (C98) and Hartman et al. (2005) (H05) in *BVI*. Strader, Everitt & Danford (2002) (S02) derived *V* time series for stars in the core of the cluster, but their light curves represent differential fluxes on an arbitrary scale, instead of standard magnitudes.

The lack of data on M3 inspired several time series studies; such as Mallik, Christensen & Saha (1999), Uglesich et al. (2000) and Arellano Ferro et al. (2002). The results of these investigations have not been published yet.

We started a multicolour $BVI_{\rm C}$ CCD survey of globular clusters at Konkoly Observatory in 1998. The main goal of the project is to obtain high quality time series for as many globular cluster variables as possible. The instrumentation enables us to observe the clusters simultaneously with two telescopes (1-m RCC and 90/60-cm Schmidt), both equipped with CCD cameras. Their different fields of view (FOVs) make it possible to obtain images with sufficient resolution for accurate photometry of both the crowded central regions and the dispersed halos, respectively. In this work we present the observations of M3.

⋆ E-mail: benko@konkoly.hu

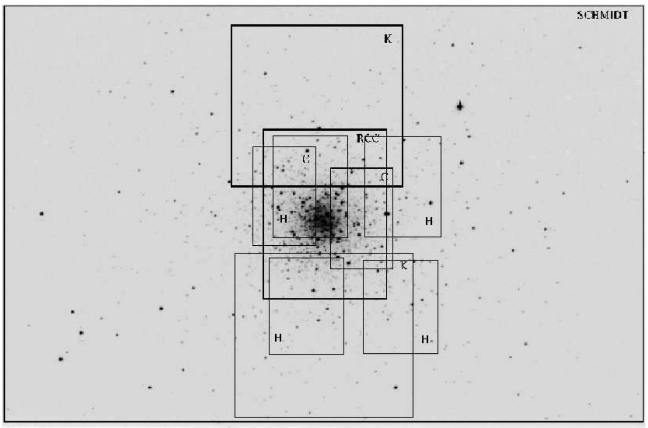

Figure 1. The relative sizes and positions of the frames used in the published CCD time series papers on M3. The notations are: 'K' – Kaluzny et al. (1998), 'C' – Carretta et al. (1998), – 'H' Hartman et al. (2005), – 'RCC' + 'SCHMIDT' – present work. Corwin & Carney (2001) have used a slightly larger FOV than 'SCHMIDT'.

2 OBSERVATIONS AND DATA REDUCTION

2.1 Observations

The observational material of M3 was obtained in two seasons (1998 and 1999) with two telescopes (see Table 1 for details). Both telescopes are mounted at Piszkéstető Mountain Station of the Konkoly Observatory. The 60/90/180-cm Schmidt telescope is equipped with a CCD camera by Photometrics Inc. having a Kodak KAF-1600 1024×1536 chip. The setup yields a $19' \times 28'$ FOV, with $1\farcs0$/pixel resolution. The 1-m RCC telescope was used with two cameras. In 1998 a camera constructed by Wright Instruments was attached to the Cassegrain focus. This device contains an EEV CCD05-20 800×1200 chip that corresponds to a $4' \times 6'$ FOV with $0\farcs35$/pixel resolution. In 1999 a Photometrics camera was used. Its basic parameters are: Thomson 7896M 1k×1k chip, $5' \times 5'$ FOV with $0\farcs29$ resolution. For the calibration process of the cameras and other technical details see Bakos (2000). Standard Johnson *BV* and Kron-Cousins $I_{\rm C}$ filters were used for all observations. From now on the band-index C is generally omitted.

The total number of images are 521 in *V*, 226 in *B* and 384 in *I*. Sky flats (or alternatively dome flats) were taken as main calibration images. Each night some bias frames (5-50) and dark images (with 3-5 minutes exposure times) were also taken.

The relative positions for the fields-of-view and the cluster for previous studies and this work are shown in Fig. 1. The FOV of our frames allowed us to get light curves from almost all variables of the cluster. The nomenclature of variables used here are the same as in the General Catalogue of Variable Stars in Globular Clusters (Clement et al. 2001, – hereafter The Catalogue).

2.2 Data reduction & photometry

We used the IRAF/CCDRED[1] package for the standard reduction procedures: bias, dark and flat field correction. Other corrections (e.g. deferred charge, non-linearity) were also investigated, but they were found to be unjustified to apply.

[1] IRAF is distributed by the NOAO, operated by the Association of Universities for Research in Astronomy Inc., under contract with the NSF.

Table 1. Log of the observations of M3.

Date	Filter / Exp. time [sec]						No.
[JD-2400000]	Schmidt			RCC			
50893		$V/300$					22
50894		$V/300$					35
50896		$V/300$		$B/300$	$V/300$	$I/180$	54
50897		$V/300$			$V/240$	$I/180$	67
50898					$V/240$	$I/180$	27
50941				$B/300$	$V/240$	$I/180$	51
50942				$B/300$	$V/240$	$I/180$	56
50943	$B/300$	$V/240$	$I/180$	$B/300$	$V/240$	$I/180$	135
50944	$B/300$	$V/180$	$I/80$	$B/300$	$V/240$	$I/180$	117
50960	$B/300$	$V/240$	$I/180$				36
50961	$B/300$	$V/240$	$I/180$				60
50970	$B/300$	$V/240$	$I/180$				46
50971	$B/100$	$V/80$	$I/60$				51*
50972	$B/100$	$V/80$	$I/60$				31*
51256				$B/240$	$V/180$	$I/180$	75
51257					$V/180$	$I/180$	50
51259					$V/180$	$I/180$	37
51262					$V/240$	$I/240$	78
51283				$B/240$	$V/180$	$I/180$	69

* Three images were combined for each data point.

The brightness of stars were determined first by using the aperture photometry task DAOPHOT/PHOT of IRAF. This method provides robust flux estimates with small errors for isolated stars, and were used for variables that lie in the outer parts of Schmidt frames (*method A*).

2.2.1 Image subtraction photometry

In the case of variable star photometry on globular clusters the Image Subtraction Method (ISM) (Alard & Lupton 1998, Alard 2000) is more efficient and accurate than the traditional PSF fitting methods, such as DOPHOT (Schechter, Mateo & Saha 1993) or DAOPHOT (Stetson 1987): more variable stars can be measured in the dense regions, and the r.m.s. of the light curves is also significantly better. We used different versions of the Alard's ISIS software package[2] as an implementation of ISM. In practice, the program was run several times by varying the most important parameters (`nstamps_x, nstamps_y, half_mesh_size, half_stamp_size, saturation, deg_spatial`) to obtain optimal subtractions. Fig. 2 demonstrates the effectiveness of the method by showing an original and a subtracted frame.

Different versions of ISIS occasionally resulted in bad photometry for several stars. Typical errors were: apparent hump(s) in the light curve, smaller amplitude than expected (amplitude errors), or unexpectedly high r.m.s. To check on these problems the IRAF/PHOT task has been run on the subtracted images. Fortunately, this simple trick has solved almost all trouble. Hence this type of photometry was also used alternatively.

The ISM method gives differential fluxes of objects with respect to their fluxes measured on the reference frame. For the instrumental magnitudes we need to determine the magnitudes and fluxes for each variable in the reference frames as accurately as possible. To get these zero points, we performed PSF photometry

[2] Available via http://www.iap.fr/users/alard/package.html.

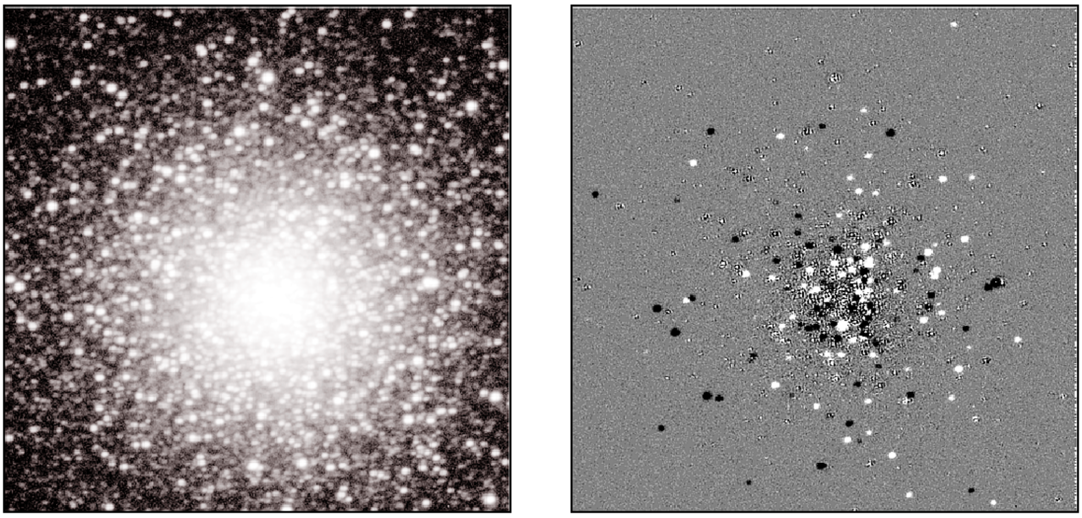

Figure 2. A typical result of the subtraction process applying the ISM method. The left panel shows the most crowded central part of a *V* frame of the Schmidt telescope, and the right panel shows the same area after subtraction. It is easy to identify the variables (white and black spots depending on whether they are brighter or fainter compared to the reference image). The other stars practically disappeared, except for few saturated ones.

using IRAF/DAOPHOT with variable PSF for the photometric reference images (one for each colour and camera). The problems of the transformation to magnitude scale have been discussed elsewhere (Benkő 2001). The zero-points were derived from either the RCC telescope's data (*method R*, in case the star was within the RCC field boundaries) or from the Schmidt frames, when it was not measured by RCC telescope (*method S*).

In the case of variables that are both on the RCC and Schmidt frames, RCC telescope data were used as a reference and the Schmidt telescope data were transformed to the RCC. In order to compute the proper zero point values the two light curves were cross-correlated by a 2D Kolmogorov–Smirnov algorithm (see Press et al. 1992). For stars with variable light curve (e.g. Blazhko effect, RRd type) light curve parts of simultaneous nights (1998 May 9 & 10) were used.

The three type of photometries (A, R, S) were tested whether they give homogeneous results or not. Our average magnitudes (see Sec. 3.1.2) were cross-correlated with CC01's ones for the stars in common. For a given colour all three methods yielded the same linear correlation: parameters of the least-squares fits agree within the regression error.

2.2.2 *Transformation into the standard system*

The instrumental magnitudes were transformed into the *BVI*$_\mathrm{C}$ Johnson-Cousins system. The standard sequence published by Stetson (2000) was used as a source of standard magnitudes. Stars that are not well separated in our frames have been omitted from these data sets. Even so, dozens of standards remained for all bands. Let us consider the colour equations as

$$\begin{aligned} V &= m_v - k'_V X + \varepsilon(B-V) + \xi_V, \\ B &= m_b - k'_B X + \mu(B-V) + \xi_B, \\ I &= m_i - k'_I X + i(V-I) + \xi_I, \end{aligned} \qquad (1)$$

Table 2. The determined telescope constants. Coefficients are defined by Eq. 1. The errors are the 1-σ of the linear least-square fits.

System	ε	μ	i
RCC '98	0.003 ± 0.01	0.202 ± 0.01	-0.040 ± 0.02
Schmidt '98	0.008 ± 0.07	-0.080 ± 0.08	-0.016 ± 0.06
RCC '99	0.004 ± 0.02	0.046 ± 0.02	-0.038 ± 0.03

where the designations are the usual: *B*, *V*, *I* are the transformed and m_v, m_b, m_i are the instrumental magnitudes, k's are the extinction coefficients, and X is the air mass, ε, μ, i are the telescope constants and ξs are the zero points. The telescope constants for the reference images have been determined by linear least-square fitting.

As the telescope constant ε in all cases turned out to be zero within the error all coefficients were calculated with the value of $\varepsilon = 0$. The obtained values are shown in the Table 2. Typical error of an individual measurement is between 0.01 – 0.02 mag in the bands *V* and *I* and 0.03 – 0.04 mag in *B*.

3 ANALYSIS AND RESULTS

3.1 Parameters of the light curves

We are presenting data here on 238 variable stars, three of them are new discoveries. The light curves of V266 are given for the first time. The construction of our observational database[3] is simple: each star has three separate files including the heliocentric Julian date (HJD) and magnitudes in the Johnson-Cousins reference *B*, *V* and *I*, respectively.

[3] Available via http://www.konkoly.hu/staff/benko/pub.html.

Table 3. Non-variable stars with variable ID in the Catalogue.

ID	B	V	I
2	15.874	15.189	14.143
98	15.387	15.593	13.327
102	16.121	15.674	14.986
103	16.565	15.930	15.026
153	14.562	13.574	12.307
164	14.730	13.818	12.633
169	15.273	14.864	13.641
182	15.967	15.454	14.726
204	15.948	15.877	15.042
227	15.995	15.469	14.769
228	16.016	15.630	15.084
231	16.074	16.000	15.851
232	15.950	15.852	15.885
233	16.224	15.935	14.680

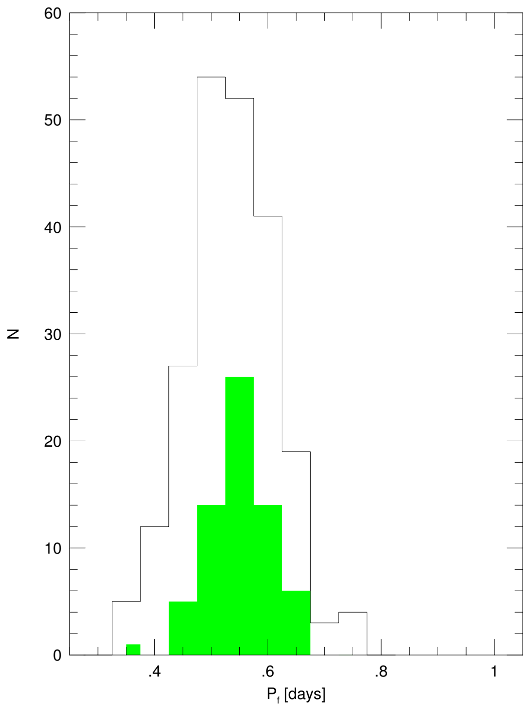

Figure 3. Histogram of fundamentalized periods ($P_{\rm f}$) of M3 RR Lyrae variable stars. Shaded bars show modulated (Blazhko type) variables which are mostly responsible for peaked distribution.

3.1.1 *Periods*

Each variable star was analyzed to find its period by using a period search on the *V* data. Discrete Fourier Transformation was applied by using the facilities of the program package MUFRAN (Kolláth 1990) or a Phase Dispersion Minimization (IRAF/PDM, Stellingwerf 1978) depending on the time distribution of collected data. Periods of the literature were accepted where the newly determined periods do not differ significantly from the old ones. The *V* phase diagrams are shown in Fig. 5.

In the period search, outlier data points were omitted from the light curves manually. Generally, those isolated points were rejected that deviated by more than 3σ from the fitted curve. In some cases, when a variable is lying near to a bright star, complete nights have to be excluded if the neighbouring bright star was saturated and therefore the ISM did not work well.

In the figure 3 we show "fundamentalized" period (for definition see van Albada & Baker 1973) distribution from our (re)calculated periods. The peaked period distribution of M3 was already obvious in figure of Oosterhoff (1939) but only Castellani & Tornambé (1981) and Rood & Crocker (1989) revealed that it is an evidence of peculiar distribution of stars within the instability strip. Catelan (2004) has reached the conclusion that this peaked period distribution might pose troubles for conventional stellar evolution theory. Assuming a bimodal mass distribution Castellani, Castellani & Cassisi (2005) obtained reasonable agreement between standard evolutionary model and observed period distribution. Therefore they do not believe in serious problems in connection with canonical models and rather prefer Catelan's suggestion: "M3 might be a pathological case that cannot be considered representative of the Oosterhoff I class". Although the evolutionary theory is beyond the scope of this paper we add further argument to latter statement in Sec. 3.3.2.

3.1.2 *Average brightnesses*

Following this, a least-square fit of a Fourier sum containing 5 – 15 sine terms was calculated.

Mean magnitudes were derived by averaging over the pulsation cycle of these fitted light curves both in intensity (then converted back to magnitude) and magnitude scales. Throughout this paper, the magnitude-averaged mean brightness is denoted by $\overline{B}$, $\overline{V}$ and $\overline{I}$, and $\langle B\rangle$, $\langle V\rangle$ and $\langle I\rangle$ symbols indicate intensity means.

This process, however, might result in erroneous values for RR Lyrae stars with Blazhko effect because Blazhko cycles have no complete coverage. Investigating Blazhko type RR Lyrae in LMC Alcock et al. (2003) have concluded that the average brightnesses varied by less than 0.006 mag over the whole Blazhko cycle. Therefore, using some subsequent nights' observations, well covered phase diagrams had been constructed for each Blazhko type RR Lyrae star, and then the average brightnesses were calculated from these phase diagrams. For about a dozen stars there was more than one possibility to make a good phase diagram. In these cases the average magnitudes were calculated from each diagram separately. By comparing the mean magnitudes of the same star at different Blazhko phases we found that the values are the same within 0.01 mag.

3.1.3 *Classification*

We measured 226 variable stars classified as RR Lyraes, 169 out of these turned out to be fundamental mode pulsators (RRab) and 48 to be first overtone pulsators (RRc). Nine variables have been found to pulsate in two modes simultaneously (RRd). We have found 64 amplitude and/or phase modulated (Blazhko effect) stars among RRab-s and 2 among RRc-s. The incidence rates of Blazhko variables are 36.7 and 4.2 per cent for RRab and RRc stars, respectively. The rate of modulated RRc variable stars is in good agreement with the values 4 – 6 per cent given by all different studies on large surveys (Alcock et al. 2000, Moskalik & Poretti 2003, Soszyński et al. 2003). Our rate, calculated from RRab stars agrees

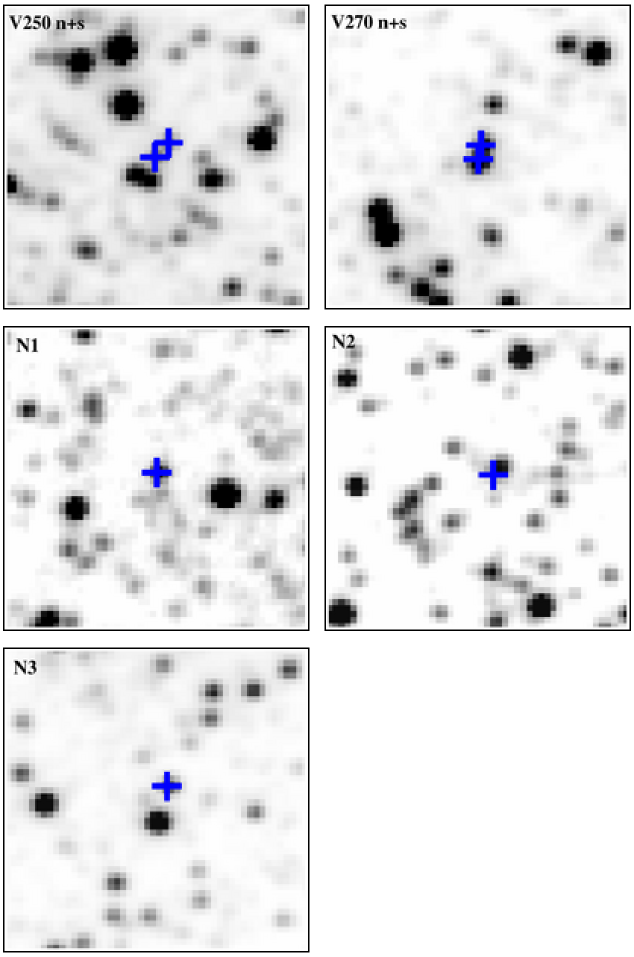

Figure 4. Finding charts for non-catalogued variables. Boxes are $\approx 15''$ in width. North is up, East is to the left.

well with that of Moskalik & Poretti (2003) as determined from the OGLE I data of the Galactic bulge. Measurements in LMC resulted in a much lower rates: 11.9 per cent from MACHO data (Alcock et al. 2003) and 15 per cent from OGLE II survey (Soszyński et al. 2003), respectively.

3.1.4 Catalogue of variables

Basic parameters of the light curves are summarized in Table 4. In column 1 the ID comes from The Catalogue. Column 2 shows the pulsation period, column 3 the type of variability, columns 4-–9 the magnitude and intensity averages. Column 10 reference letters indicate the type of magnitude transformation: 'A', 'S' and 'R' (see Sec. 2.2 for the details). In the last column comment 'Bl' indicates Blazhko type variability, 'm' merging with nearby variable or a saturated bright star and 'c' extra comments in the text. The second (non-dominant) period of double mode RR Lyrae is also shown in this column. When our data have no complete phase coverage, the average brightnesses were derived by using all published data shifted to our ones. Comment 'a' refers to this. The colon at an average magnitude indicates noisy or gappy light curve and at a reference letter it means a doubtful zero point of the magnitude transformation.

Numerous stars can be found in The Catalogue, which turned out to be non-variable. We summarized our observations on these constant stars in Table 3. It has to be mentioned that even the newest on-line version of The Catalogue[4] contains position errors. The column declination must have been shifted accidentally by one line at the variables V238, so the declinations of V238, V239 are wrong. In the Table 5 the correct positions are repeated to avoid any further confusion.

Table 5. Errata of the positions of Clement et al. (2001), position of new variables and non-variable stars found in the instability strip

ID	$\alpha(2000)$ [h:m:s]	$\delta(2000)$ [$\circ : \prime : \prime\prime$]	type
V238	13:42:15.75	28:18:17.2	EW
V239	13:42:09.87	28:22:15.9	RRab
V240	13:42:09.46	28:22:35.3	RRc
N1 = NV291	13:42:18.06	28:22:40.1	SXPhe
N2	13:42:12.41	28:22:34.4	RRc
N3 = NV292	13:42:11.17	28:21:54.1	RRc
V250n	13:42:10.48	28:22:52.9	RRab
V250s	13:42:10.54	28:22:52.1	RRab
V270n	13:42:11.95	28:23:32.7	RRab
V270s	13:42:11.96	28:23:31.9	RRab
vZ525	13:42:07.70	28:21:52.9	
	13:42:12.64	28:22:11.4	
	13:42:01.85	28:22:36.1	
	13:42:10.46	28:22:47.4	

3.2 New variable stars

During our preliminary data processing six new variable stars have been discovered and published elsewhere (Bakos et al. 2000). We searched for additional new variable stars in 'variability images' prepared by ISIS. Scanning these images three additional new variables have been identified. They are designed as N1, N2 and N3 (for finding charts see Fig. 4). To verify their positions and light curve parameters turned out that N1 and N3 are the same stars as NV291 and NV292, respectively in the meantime published study of Hartman et al. (2005). The colour indices, period and light curve shapes suggest that N2 is a new RRc type star.

When the images of two (or more) variable stars are blended their light curves became confusing. This was the situation in the case of the light curves of V250 and V270 (see Fig. 5), albeit no close companions were known to them. By inspecting the subtracted images we have found several ones in both cases in which two close variable stars can be separated. The blended light curves suggest that in both cases two RRab stars are merged. We have split their notations into two as V250n, V250s and V270n, V270s as in the similar situation of V4. Positions of all new variable stars are given in Table 5, and finding charts in the Fig. 4

After the appearance of the latest version of The Catalogue some suspected new variables were reported in S02. We scanned our images to confirm their discoveries, but it was not successful in any cases.

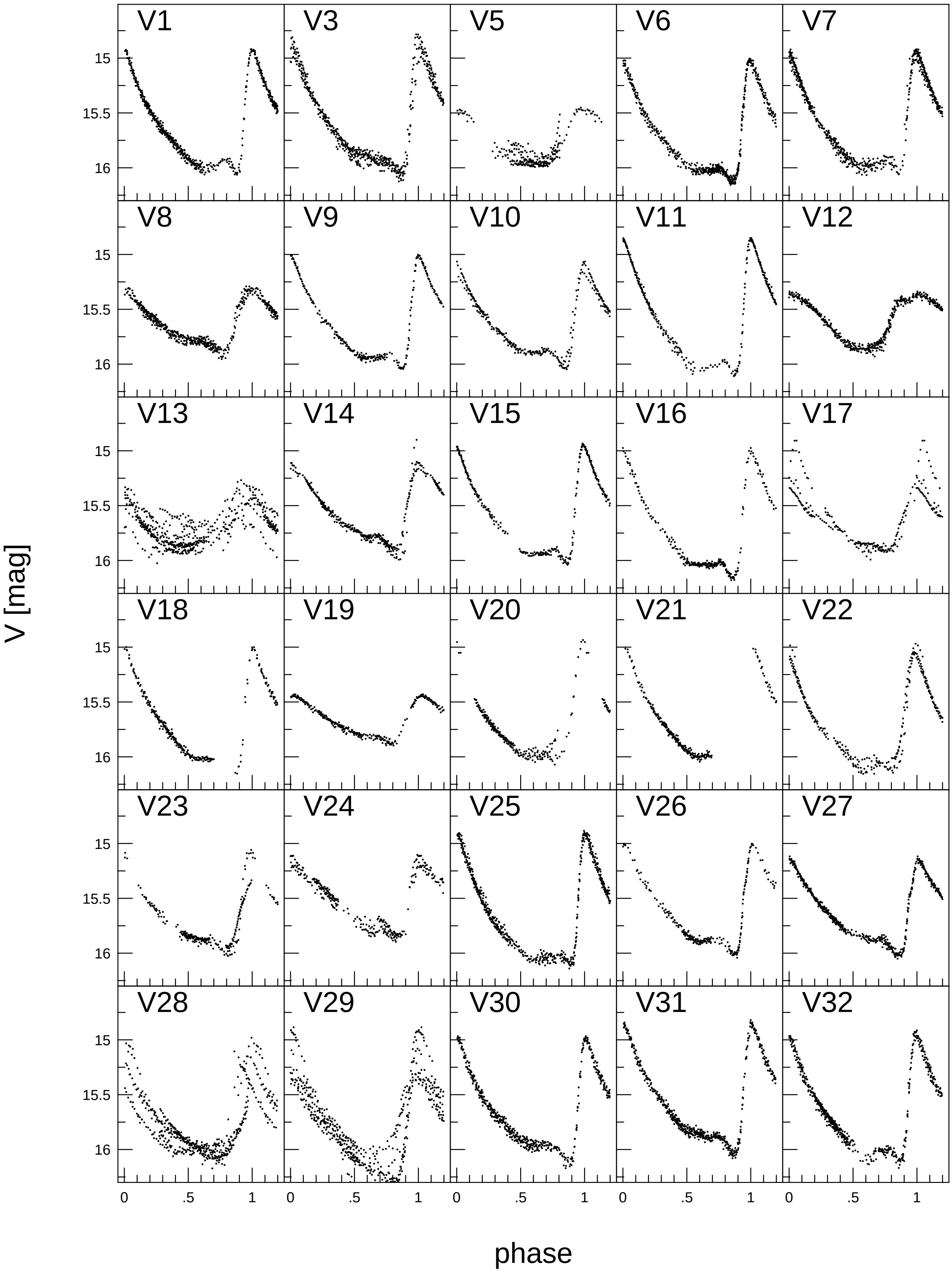

Figure 5. The V phase diagrams of the measured variables. This figure is published in its entirety in the electronic edition of this journal

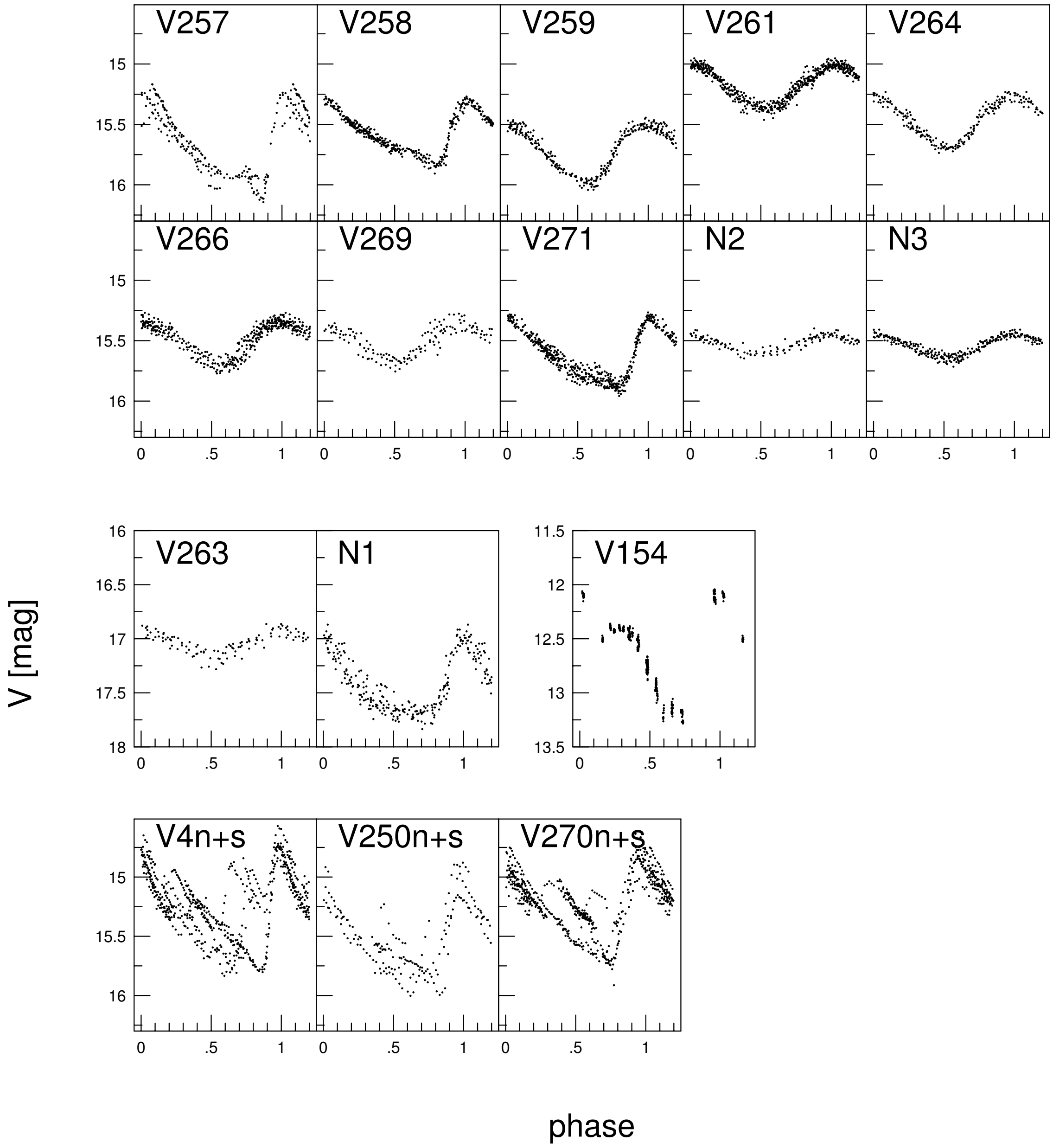

Figure 5. (continued)

Table 4. The basic parameters of the variables in M3. Table 4 is published in its entirety in the electronic edition of this journal

ID	Period	Type	$\overline{B}$	$\langle B\rangle$	$\overline{V}$	$\langle V\rangle$	$\overline{I}$	$\langle I\rangle$	$\mathrm{Ref}_{B,V,I}$	Comm.
1	0.5205963	RRab	16.004	15.900	15.688	15.631	15.248	15.227	R R R	3
3	0.5581979	RRab	15.939	15.857	15.661	15.612	15.175	15.155	R R R	Bl,3
4n	0.585029	RRab							R R R	c
4s	0.593069	RRab							R R R	c
5	0.505703	RRab	15.986	15.974	15.725	15.714	15.243:	15.238:	A A S	Bl,c
6	0.5143327	RRab	16.083	16.003	15.755	15.703	15.294	15.276	R R R	1
7	0.4974248	RRab	15.980	15.889	15.694	15.638	15.248	15.230	R R R	Bl,2
8	0.636728	RRab	15.808	15.787	15.649	15.634	14.902	14.898	R: R: R:	3
9	0.5415553	RRab	16.031	15.957	15.689	15.645	15.233	15.217	A A S	2
10	0.5695465	RRab	16.057	16.004	15.684	15.647	15.162	15.152	S S S	Bl,2,c
11	0.5078915	RRab	15.993	15.873	15.706	15.636	15.188	15.166	S S S	3
12	0.3179347	RRc	15.840	15.814	15.621	15.606	15.281	15.275	R R R	
13	0.4795043	RRd	15.931	15.917	15.691	15.684	15.243	15.239	R R R	$P_1 = 0.351579$
14	0.6359002	RRab	15.914	15.867	15.581	15.551	15.059	15.050	R R R	Bl,4
15	0.5300874	RRab	15.919	15.833	15.667	15.616	15.159	15.144	A A A	3

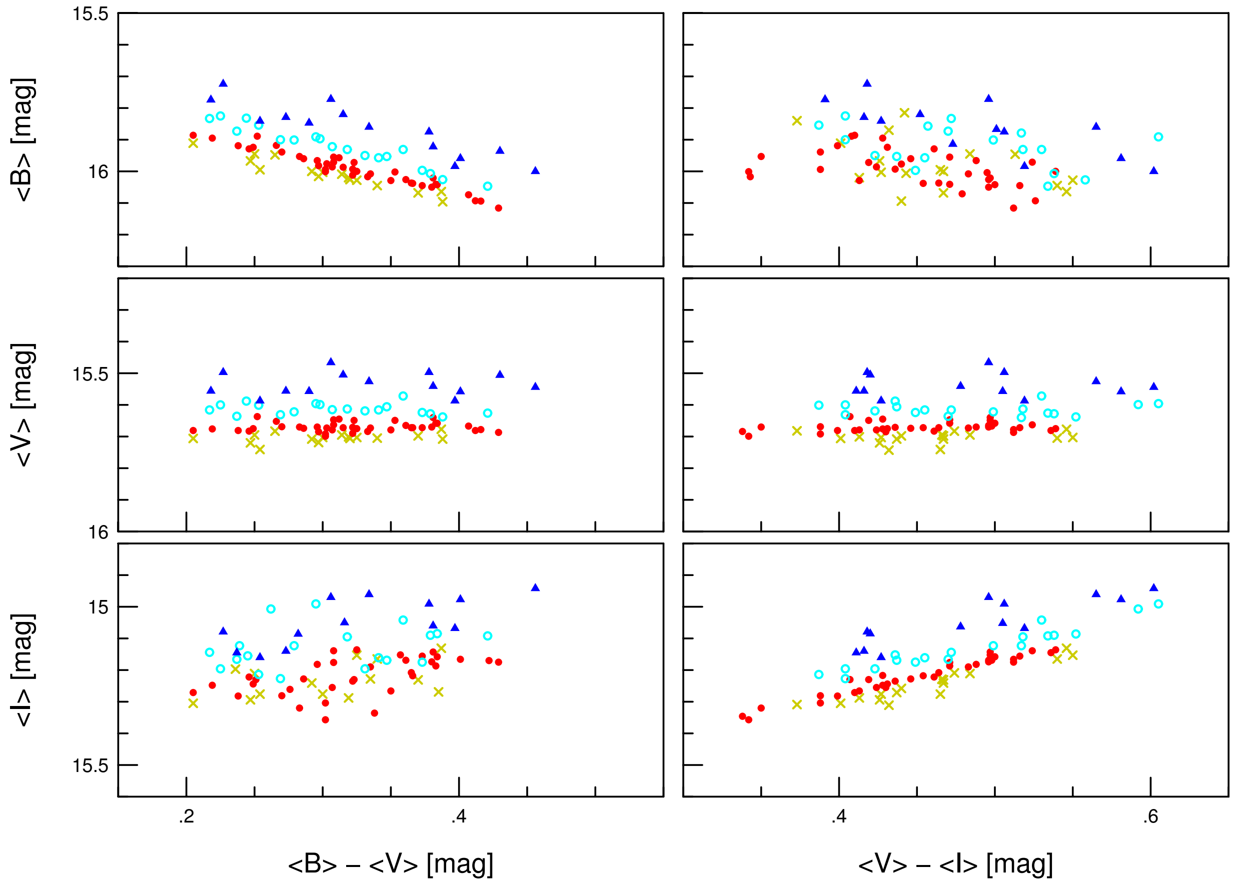

Figure 6. Correlations among colours and color-indices of single mode RRab stars in M3. Yellow times crosses, red dots, blue open squares and filled blue squares denote the same luminosity/evolutionary subgroups as separated by Jurcsik et al. (2003) among RRab variables.

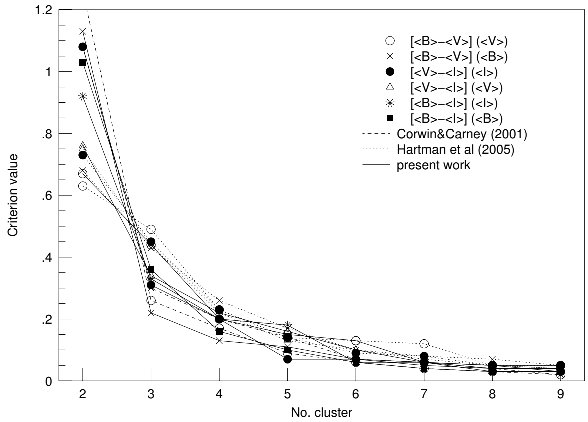

Figure 7. Criterion value in the function of the number of clusters determined by cluster analysis of CMDs. Different observational sets and CMDs are encoded by line and dot styles mentioned in the figure. The cut-off points of functions suggest three or four intensity groups.

Table 6. Connections between the found *clusters* by cluster analysis and *intensity groups* defined by Jurcsik et al. (2003) in the CMDs assuming different number of groups/clusters.

group ID[1]		No. stars		CMD type
	group[2]	cluster	common	
1+2+3	76	76	75	$[\langle B\rangle - \langle V\rangle](\langle V\rangle)$
1+2	57	59	55	
3	19	17	14	
4	14	14	13	
1+2+3	73	72	69	$[\langle V\rangle - \langle I\rangle](\langle V\rangle)$
1+2	53	54	48	
3	20	18	10	
4	13	14	10	
1+2+3	73	62	57	$[\langle V\rangle - \langle I\rangle](\langle I\rangle)$
1+2	53	49	47	
3	20	13	10	
4	13	24	8	
1+2+3	76	67	51	$[\langle B\rangle - \langle V\rangle](\langle B\rangle)$
1+2	57	54	42	
3	19	13	9	
4	14	23	8	

[1]The members of the groups are signed by these ID-s in Table 4.
[2]The number of these stars are differ in 1-2 from the number of stars given by Jurcsik et al. (2003), because we used only those stars which have good phase coverage from our observations themselves.

3.3 Single mode RR Lyrae stars

3.3.1 Mean magnitudes

As Marconi et al. (2003) have shown the intensity-averaged mean magnitudes of RR Lyrae stars are approximated well by the physically relevant static brightness. The difference between static and mean magnitudes depends on the amplitude but the maximum value of this correction would be only ~ 0.02 mag for the highest amplitude variable star in our sample of M3 (V148 at $A_V = 1.38$ mag). Hence, throughout this section we investigated intensity-averaged mean magnitudes without corrections.

Cacciari, Corwin & Carney (2005) (CC05) have compared the intensity-averaged magnitudes among data from CC01, C98 and K98 in details. Observations in *B* and *V* bands from CC01 data were found significantly brighter than from C98 and a slightly brighter than *V* data from K98. We found our $\langle V\rangle$ and $\langle B\rangle$ magnitudes of common 92 non-modulated RR Lyrae stars to be 0.018 and 0.013 mag fainter then those of CC01. As CC05 pointed out the zero point of $\langle I\rangle$ magnitudes of C98 is too faint. On the basis of 46 RR Lyrae stars in common with C98 data sets the zero point shift between our and C98 data is -0.044 mag while CC05 estimated the zero point shift between standard magnitude scale and C98 one's from the reddening calibration formula as -0.08 mag. We performed a similar comparison between H05's data and ours. Data from H05 proved to be brighter than ours for bands $\langle V\rangle$ and $\langle I\rangle$ with 0.017 and 0.016 mag and 0.008 mag fainter in band $\langle B\rangle$.

These differences are certainly related to the long standing problems of absolute calibration.

3.3.2 Colour – magnitude diagrams

As it is well known the colour – magnitude diagram (CMD) of the horizontal branch of globular clusters shows, in general, a good separation between variable and non-variable stars. Hence, we have investigated the instability strip of the horizontal branch in the $[\langle B\rangle - \langle V\rangle]$ $(\langle V\rangle)$ and $[\langle V\rangle - \langle I\rangle](\langle I\rangle)$ CMDs simultaneously to check the completeness of variables' list. However, no additional variable stars have been found, but four non-variable stars seem to be located in the instability strip.

To decide whether these stars are members of the cluster or not, the literature (Cudworth 1979, Tucholke, Scholz & Brosche 1994) was looked through to get proper motions of these stars but unfortunately, no measurements have been found. These stars are most likely in the foreground. But if any of them proved to be a member of the cluster, it would be a serious challenge to the pulsation theory. The position of these problematic stars is given in Table 5. CC01 also reported eight constant stars in the instability strip of their *B*−*V*, *V* CMD, but without position, therefore, we could not compare them with our ones.

The separating lines between the $\langle B\rangle - \langle V\rangle$ colour for variable and non-variable stars are ~ 0.16 mag at the blue edge of the instability strip and ~ 0.43 mag at the red edge. The dividing line between the RRab and RRc variables is about 0.21 mag although this separation is less definite than in magnitude-averaged magnitudes. The borders of the instability strip in the $[\langle V\rangle - \langle I\rangle](\langle V\rangle)$ CMD are 0.25 mag and 0.55 mag. While the dividing line between fundamental and overtone pulsators is about 0.35 mag. Comparing these values with previous studies we found that both the blue and red edges of our instability strip are slightly bluer than from data of H05, but similar to C98, CC01. The width of instability strips are, however, the same (~ 0.27 mag) for all three cases.

Jurcsik et al. (2003) have studied relations in $\langle V\rangle$ mean magnitudes and light curve parameters (viz. period and Fourier coefficients). Four subgroups were separated from non-modulated RRab stars according to their brightness and light curve shapes and these groups were interpreted as different stages of horizontal branch stellar evolution. The brightest group shows Oosterhoff II properties as opposed to the other three ones. This result suggests that the Oosterhoff dichotomy of the Galactic globular clusters is to

[4] Available at http://www.astro.utoronto.ca/~cclement/cat/listngc.html.

be an evolutionary effect and M3 is not a typical Oosterhoff I cluster. In Table 4 these groups of RRab stars are marked with number 1-4 from the faintest group to the brightest one. Recently CC05 have questioned the existence of these four groups. They found a bimodal structure in the magnitude distribution: a brighter group at $\langle V \rangle = 15.52 \pm 0.02$ (which is in the same position as the brightest group of Jurcsik et al. 2003), and a main body at $\langle V \rangle = 15.64 \pm 0.04$.

Through this subsection we have reanalysed those non-modulated RR Lyrae stars which were selected by Jurcsik et al. (2003) on the basis of their good quality light curves. The $[\langle B \rangle - \langle V \rangle](\langle V \rangle)$ and $[\langle V \rangle - \langle I \rangle](\langle V \rangle)$ colour magnitude diagrams in Fig. 6 correspond to fig. 4 in the paper Jurcsik et al. (2003). The similar structure of figures can be clearly recognized: the four subgroups of RRab stars with different brightness are lying in parallel strips, and the averaged periods are shifted from shorter to longer according to the brightness changes from the fainter to the brighter. The same structures appeared in all other diagrams in Fig. 6. Naturally, the parallel strips have different steepness for various colours. The groups in $[\langle B \rangle - \langle V \rangle]\,(\langle I \rangle)$ and $[\langle V \rangle - \langle I \rangle]\,(\langle B \rangle)$ CMDs are less evident compared to the other ones caused most probably by higher scatter resulting of measurements in the three independent bands.

We have examined all colour – magnitude relations with the cluster analysis, an efficient multivariate statistical method, to point out the solidity of identification of these groups. (We refer here to the book of Murtagh & Heck (1987) as an excellent introduction to the method.) The Ward's minimum variance method was chosen from the many different clustering algorithms which is generally a very robust process. (It should be mentioned, that the average linkage method has also fit well for the data and yielded very similar results.) The algorithm is a hierarchical type clustering which does not assume any prior knowledge about the number of clusters. Finding the right number of clusters is based on a horizontal "line draw" through the resultant hierarchy represented by a dendrogram. The choice of where to draw the line is derived from large changes in the criterion value.

Fig. 7 shows the criterion value – cluster number functions obtained from different standardized data sets where in addition to our data CMDs were also used from CC01 and H05 of stars in common with our sample. Most functions belonging to different data sets have a well defined cut-off point where the criterion value dropped. The $[\langle B \rangle - \langle V \rangle](\langle V \rangle)$ ratios derived from CMDs, based on H05 and our data set, suggest *four clusters*. Other CMDs obtained from CC01 and present observations seem to segmented into *three clusters*. The cut-offs are less specific at the functions of H05 data because of the small number of common stars (49-59 depending on colours).

To investigate the connection of signed groups in Figures 6, 8 – 9 and groups found by the cluster analysis the star content of each cluster has been checked and the result are given in Table 6. *a)* When we assumed two subgroups according to CC05 and groups signed with 1, 2 and 3 by Jurcsik et al. (2003) were handled together while group 4 remained separated the two clusters obtained from cluster analysis corresponded well to these sets both in number of elements and star content. In the case of $[\langle B \rangle - \langle V \rangle](\langle V \rangle)$ CMD both analyses yielded 76 and 14 members clusters with 75 and 13 overlapping of star contents, respectively. Similar connections can be realised for the $[\langle V \rangle - \langle I \rangle](\langle V \rangle)$ CMD but less overlapping star contents. Using elementary formulae of statistics the probabilities of such a connections are $p = 1.17 \cdot 10^{-13}$ and $p = 4.8 \cdot 10^{-12}$, respectively. Overlapping structures for the other

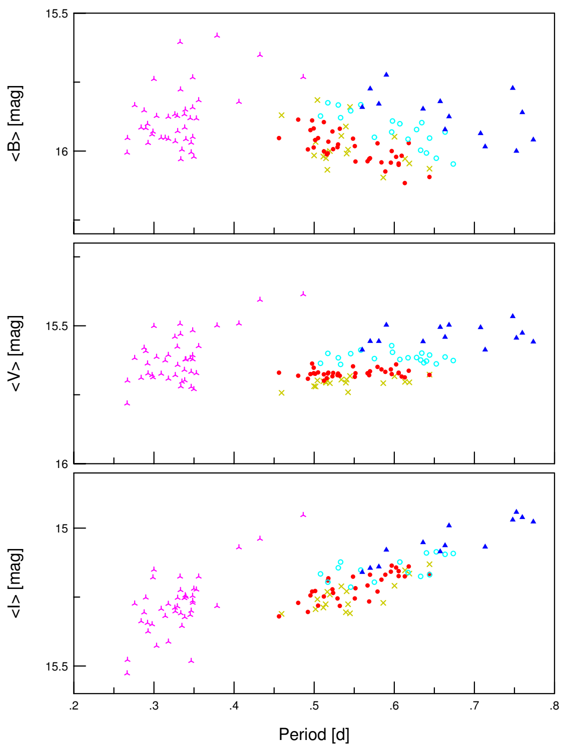

Figure 8. Intensity-averaged mean magnitudes of non-modulated RR Lyrae stars vs. period. Symbols as in Fig. 6, the purple three-prong crosses denote RRc variable stars.

two CMDs are not statistically significant. *b)* However, our cluster analysis suggested at least three clusters (Fig. 7). In the models with three sets (groups 1+2, 3 and 4 vs. three clusers) $[\langle B \rangle - \langle V \rangle](\langle V \rangle)$ and $[\langle V \rangle - \langle I \rangle](\langle V \rangle)$ CMDs yielded again satisfactory result. The probability that these structural similarities are only incidental is very low $p = 6.2 \cdot 10^{-15}$ and $p = 3.4 \cdot 10^{-14}$. *c)* No significant connections were found between four groups and four clusters models.

As a conclusion of this section we say that our cluster analysis on CMDs found the same two sets of stars as they were found on the basis of Bailey diagram and period distribution by CC05 and the same three sets which were identified by Fourier parameters in Jurcsik et al. (2003).

3.3.3 Period – brightness and period – colour relations

In the panels of Figure 8 the magnitudes vs. period are plotted where the four above discussed luminosity groups are indicated with the same symbols as in Figure 6. The highest scatter of plot $P(\langle B \rangle)$ is originated probably not only on account of the higher noise level of observations, but due to the fact that the effects of luminosity and temperature variations upon the expected periods are almost orthogonal in this plane (see Catelan, Pritzl & Smith 2004).

In the panels of Fig. 8 RRab and RRc stars are well separated, except for four variables: V70, V129, V170 and V261. Most likely, these variables belong to a distinct group on the basis of their common properties. They have asymmetric light curves with amplitude

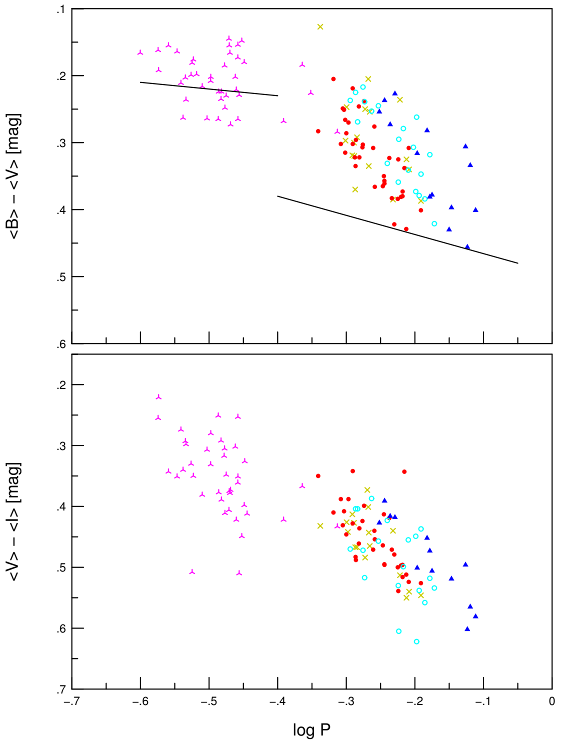

Figure 9. Colour indices vs. period. The lines indicate the predicted edges of instability strip taken from Marconi et al. (2003) at the mixing length parameter of 1.5. Other symbols are as in Fig. 8

between RRab and RRc variables. Their periods are also between those of the two groups and mean brightnesses for all filters are higher than the other RR Lyrae stars at a given period. V70, V85, V129, V170 and V177 have been characterized by CC05 as "long P / overluminous" stars. H05 and present data show V85 and V177 to be regular RRc stars.

A definite correlation can be seen between period and magnitude $\langle I\rangle$ of RRab stars. Applying a linear least-square fit for 76 non-modulated RRab variables, we have found

$$\langle I\rangle = -1.453 \log P + 14.828, \quad (2)$$

with $rms = 0.04$ mag. To our best knowledge the work of Layden & Sarajedini (2003) was the first paper in which a period-luminosity type relation was derived in the I band for the RR Lyrae stars in NGC3201. Pritzl et al. (2003) discussed this relation in the case of the globular cluster NGC6441. Catelan et al. (2004) demonstrated that this type of relations must exist. They plotted period–luminosity type graphs in standard $UBVRIJHK$ photometric bands, for some different metallicities and horizontal branch morphologies (evolutionary states) based on synthetic horizontal branch calculations. Fig. 6 is qualitatively similar to figs 6-9 in the paper of Catelan et al. (2004). In addition, we derived the period–luminosity relation for M3 with the formula given by Catelan et al. (2004) accepting the parameters Lee-Zinn type horizontal morphology index and metallicity as 0.282 and $Z = 0.001$, respectively. The result is $M_I = -1.644 \log P - 0.234$ which is also close to the observed relation Eq. (2).

Table 7. Estimated modulation periods P_m of RR Lyrae stars with Blazhko effect. The sources of used data and total number of observed nights are also indicated. The slight phase modulations are noted by 'phase' in column remark.

ID	No. nights	ref.	P_m [day]	remark
V66	59	1,2,3,4	56.24	
V67	59	1,2,3,4	80.94	
V10	52	1,2,3,4	124.78	
V43	51	1,2,4	73.54	
V78	51	1,2,4	38.65	
V121	51	1,2,4	64.37	phase
V104	50	2,3,4	63.78	
V59	43	2,3,4	62.14	
V63	43	2,3,4	44.91	
V101	41	2,4	91.15	
V110	41	1,2	45.66	
V7	40	2,4	61.9:	
V47	40	1,2	212.06:	phase

1. Carretta et al. (1998), 2. Corwin & Carney (2001), 3. Kaluzny et al. (1998), 4. Hartman et al. (2005)

We have to mention that no clear correlations can be identified for RRc variable stars in contrast to RRab ones. However, converting the periods of RRc variables to their fundamental mode equivalents (P_f) RRc stars fit well to the period – $\langle I\rangle$ relationship extending it to shorter periods. What is more, fundamentalized periods of overtone pulsators define the same relation within the fitting error ($rms = 0.07$ mag) as Eq. (2): $\langle I\rangle = -1.486 \log P_f + 14.748$. The range of period for the RRc stars is smaller than for RRab, so the relation between period and $\langle I\rangle$ is naturally less evident and noisier than from RRab stars but it is even so significant.

In Figure 9 the log P – colour relations are plotted. To harmonize CC01 data with the theoretical models Marconi et al. (2003) assumed an increasing in the mixing-length parameter from 1.5 to 2.0 from the blue edge to the red one. In the upper panel of the Fig. 9 the predicted edges of the instability strip at the mixing-length parameter $1/H_p = 1.5$ (taken from the paper Marconi et al. 2003) are also shown. On the basis of our data also an increasing mixing-length parameter seems to be adequate but varying form ~ 1.1 to ~ 1.5. Considering interstellar reddening the situation does not change much because the reddening is $E(B-V) = 0.01 \pm 0.01$ for M3 obtained by different methods (for a review see CC05).

3.4 Variable stars with Blazhko effects

The amplitude and/or phase modulation of several RR Lyrae stars was discovered by Blazhko (1907), but the correct explanation of the effect is still missing (see Dziembowski & Mizerski 2004 for present situation). Since M3 contains the highest number (~ 70) of RR Lyrae stars in one cluster showing Blazhko effect, this offers a unique opportunity to investigate this interesting phenomenon. Unfortunately, the typical accuracy of the earlier photographic observations (see for a review and complete references Szeidl 1965, 1973, Kukarkin & Kukarkina 1961, 1970, Welty 1985) were inadequate to studying the Blazhko effect in details. A basic parameter such as the period of the Blazhko cycle was determined only for one star (V5 by Panov 1980). On the other hand, high quality CCD observations are badly sampled and they typically consist of about a dozen of nights spreading over 1-3 years.

Studies based on the large surveys (MACHO: Alcock et al.

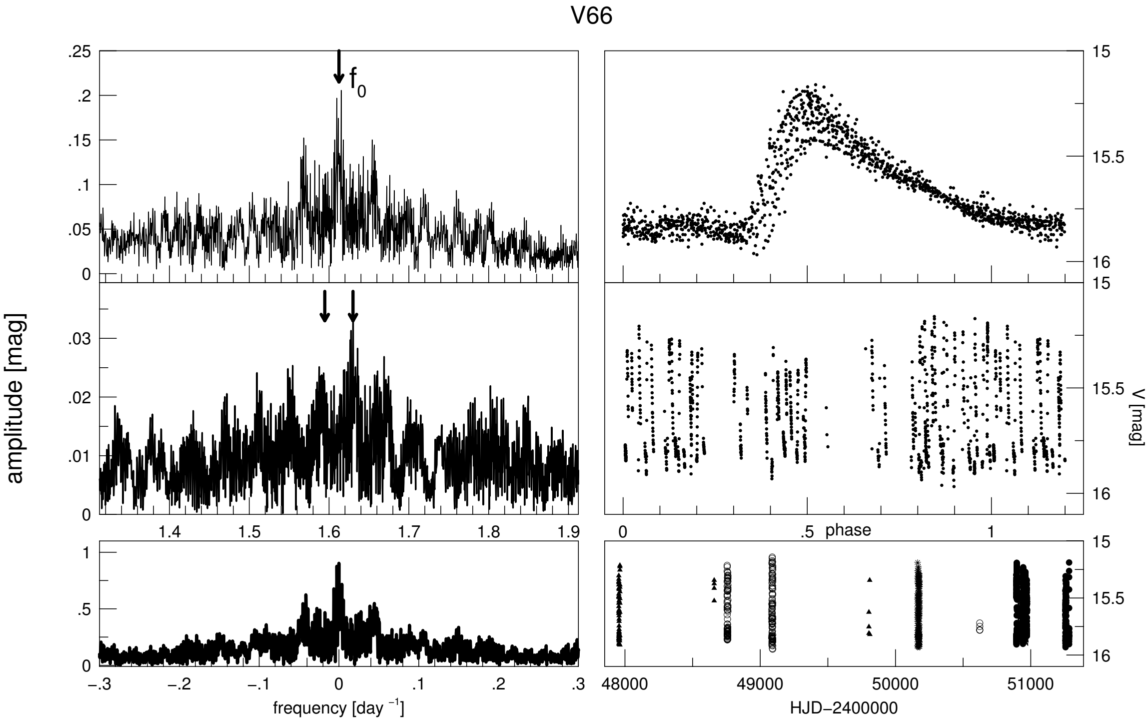

Figure 10. An example for Blazhko period finding. Figures for all stars in Table 7 are available in electronic form. Combined *V* light curve folded with the best pulsation period (top right). Fourier amplitude spectrum of combined data around the strongest peak signed by f_0 (top left). Residual spectrum of prewhitened data with $kf_0, k = 1, \ldots, 10$, where the $f_0 \pm f_m$ frequencies are noted by arrows (middle left). Light curve folded with modulation period (middle right). Data distribution (bottom right), where the symbols denote sources viz. triangles: C98, open circles: CC01, stars: K98, crosses: H05, dots: this work. The spectral window is shifted to f_0 in the same scale as the above amplitude spectra (bottom left).

2000, 2003 and OGLE: Moskalik & Poretti 2003) distinguish among different type of modulations based on the structure of the Fourier spectrum of stars. As Jurcsik et al. (2005a,b) have expressed this classification likely resulted in too many subtypes. The classical BL-type (frequency triplet with two equidistant, side lobes around the main frequency) and type ν_1 (frequency doublet) may differ only in their data distributions: sampling, S/N ratio. Since the number and time coverage of our data are rather limited these subtypes were not separated and both are called as Blazhko modulation.

The light curves of all observed RR Lyrae stars were searched for modulation. Revising the literature from point of view of Blazhko behaviour some controversial cases were found. In the cases of V10, V22, V54, V44, V140 and V218 the Blazhko modulation was verified. In the case of V18 we cannot detect any light curve variations on the basis of all CCD observations in contrast to earlier photographic studies. In addition, we found evident light curve changes for V104, V168, V172, V191 and V211 for the first time. Details of individual stars are discussed in Sec. 3.6.

Our technique for obtaining average brightnesses of Blazhko stars is described in Sec. 3.1.2. The distributions of Blazhko type RR Lyrae in the period – luminosity diagrams are similar to the non-modulated variables, but their numbers are significantly less at longer periods (higher brightnesses). Alcock et al. (2003) found a similar effect in the case of LMC variables and CC05 demonstrated it in the case of M3 as well.

3.4.1 *Search for Blazhko period*

The period of Blazhko cycle is an important parameter of these stars. Since the typical modulation period is about 10-500 days (Smith 1995), it cannot be determined from a data set optimized to pulsation cycles of RR Lyrae stars. To determine Blazhko period all CCD *V* data of all modulated RR Lyrae stars in the cluster were collected. In preparing homogeneous data sets we had to take into account the zero point shifts among different photometries which differ from star to star. Jurcsik et al. (2005a) have demonstrated that the light curve changes are concentrated in a narrow phase interval ($\sim$ 0.2) of pulsation: 0.4 – 0.6, if maximum is at 0.5. We confirmed this result for all RR Lyrae stars in M3 with pure amplitude modulation. The residual phase diagrams folded with pulsation period after prewhitening with the mean pulsation light curves clearly showed the narrow range of variability. Strong phase modulation, however, removed the effect.

Applying this result we prepared phase diagrams for each Blazhko type star from each pair of data sets containing the present and one of the published data sets. Phase diagrams were divided into 10-15 bins, then the *average of standard deviations of the bins* (ASDB) was calculated so that bins around the maximum phase were omitted. We searched for the *minimum* of the ASDB while data set taken from the literature was shifted by 0.01 mag at each iteration. The shifting value at the minimum of the ASDB yielded the proper zero point to combine a given pair of data sets. The process was carried out for each star and pair of data sets.

Figure 10 demonstrate the process of modulation period search. The spectra were computed by discrete Fourier algorithm included in MUFRAN program package by Kolláth (1990). The combined data were Fourier analyzed to refine the pulsation frequency. The prewhitened spectrum (with the pulsation frequency and its harmonics) was searched to find close frequenci(es) to the main one. To verify the estimated modulation frequency window spectrum was compared with the spectrum of data and light curve folded with modulation period was also prepared.

The results of this period search are shown in Table 7. For 13 variable stars a more or less established value of modulation period has been found for the first time. Length, distribution and/or quality of data of other stars were not sufficient for finding a modulation period. Let us mention here that just four stars (V10, V34, V66, V67) are included in all published CCD surveys (C98, K98, CC01, H05 and present work)! Only 37 stars were observed at least in 40 nights by CCD which amount of observations proved to be necessary to estimate a Blazhko period. The found periods are within the normal range of Blazhko cycles and far below the borderline of limit periods discovered by Jurcsik et al. (2005b).

Some words about modulated overtone pulsators V140 and V168. There were found frequency triplets/doublets at numerous RRc stars in different globular clusters (Olech et al. 1999a,b, 2001, Walker 1994, Clement & Rowe 2000), but no previous studies reported modulated RRc stars in the case of M3. As it was stressed above combining of different photometries works properly only for pure amplitude modulation. V140 and V168 have strong phase modulations and their amplitude changes are only marginal so their different photometric data sets had to be analyzed separately. In the Fourier spectrum of prewhitened data of V140 two peaks and their harmonics appeared equidistantly from the main frequency and its harmonics respectively, indicating a modulation period about 9 – 14 days. However, consistent period could not be found. In contrast to V140 the residual spectrum of V168 shows only one peak and its harmonics at $f_1 = 3.4754158$ day^{-1}. If we interpret it as $f_1 = f_0 - f_m$ modulation frequency the Blazhko cycle would be 6.743 days. An alternative explanation for the peak is excitation of a non-radial mode as it was discussed by Olech et al. (1999a,b).

3.5 Double mode RR Lyrae stars

Presently 74 galactic globular clusters are known containing RR Lyrae stars (Clement et al. 2001). About 3 per cent of the total number of ~ 2000 cluster RR Lyrae stars are double mode (RRd) pulsators. Their distribution among clusters is, however, highly non-uniform: only six clusters (NGC2419, M68, M3, IC4499, NGC6426 and M15) contain at least one RRd stars. The richest clusters in double mode RR Lyrae stars are M68 (12 RRd, 29 per cent), M15 (17 RRd, 19 per cent) and IC4499 (17 RRd, 18 per cent). The rate of RRd stars of M3 (9 RRd, 3 per cent) does not exceed the averaged rate. But it is still noteworthy since we know clusters overall rich in RR Lyrae stars but no RRd stars were detected in them (e.g. ω Cen, M62, M5).

A series of studies (Nemec & Clement 1989, Clement et al. 1997, Corwin, Carney & Allen 1999, Clementini et al. 2004 (CC04), and further references therein) revealed double mode RR Lyrae content of M3. In these studies eight stars were identified as RRd variable: V13, V68, V79, V87, V99, V166, V200 and V251. Hartman et al. (2005) have suggested additional four double mode candidates: V252, V253, V270 and NV290. They stressed that the multi-periodic behaviour of these stars were determined by eye without systematic search so we have investigated light curves and Fourier spectra of the first three candidates. V252 proved to be an

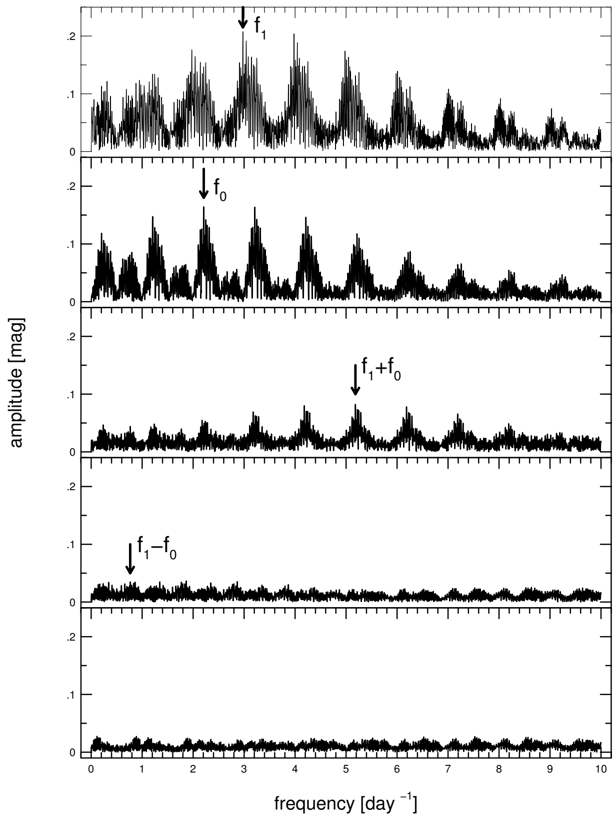

Figure 12. Fourier amplitude spectra for the RRd variable V252. The frequency searching is demonstrated step by step from top to bottom panels.

RRd variable indeed (see the following section). As it was mentioned before, the strange brightness variation of V270 is caused by two overlapping RR Lyrae stars and V253 seems to be a normal RRc star.

3.5.1 *V252 as a new RRd star*

From our light curves it turned out that V252 shows double mode behaviour. The uppermost panel in Fig. 12 presents the amplitude spectrum of the *V* data. The next panel shows the spectrum after prewhitening with the dominant frequency $f_1 = 2.97628$ day^{-1} and its six harmonics. The third panel is a plot of the prewhitened spectrum with $f_0 = 2.208528$ day^{-1} and its seven harmonics. In this spectrum the linear combinations of the two radial modes $f_1 + f_0$ can clearly be identified and we get the fourth spectrum after that we whitened out with it. The $f_1 - f_0$ linear combination is much less significant than $f_1 + f_0$ in the previous spectrum, however, considering this odds the r.m.s. of the least square fitted light curve has been improved well. The lowest panel shows the final whitened spectrum. No additional peak can be recognized. Fig. 11 was obtained so that the light curve was fitted simultaneously by all frequencies obtained from the above process. The 1σ error of the fit is 0.04 mag which is slightly larger than the intrinsic observational error indicating other linear combinations and/or higher harmonics in the data. Because of their low amplitude it is impossible to find them from these data.

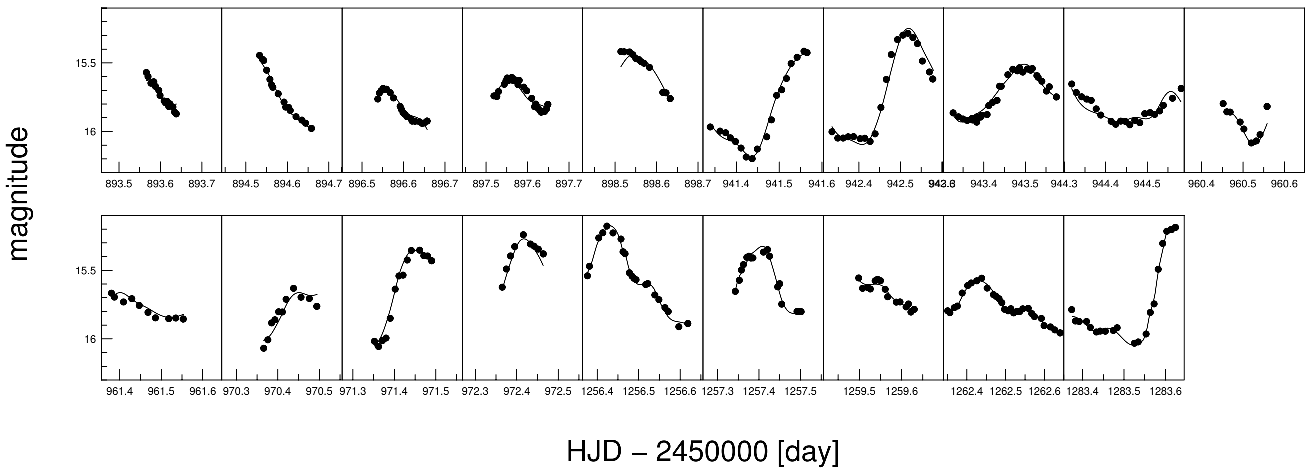

Figure 11. Observed and fitted *V* light curve of the new RRd variable V252. Each night was plotted on the same size of time interval.

3.5.2 *Period and period ratios*

The periods of all measured double-mode variables were determined as it was described for V252 (see Tab. 4 for the values). The periods determined by us are generally in good agreement with the values given by CC04. The period of fundamental mode (P_0) of V79 started to decrease drastically in the middle of the last century (Clement & Goranskij 1997) and we found it still decreasing. The collected data from the two recently recognized RRd stars V200 and V251 allowed us to specify their periods. The new periods and period ratio of V200 have placed it among the ‘normal’ double-mode RR Lyrae stars as opposed to the work of CC04. However, the Petersen diagram of the cluster in Fig. 13 shows that V13 is very far from all other RRd stars. What is more, V13 has the smallest period ratio (~ 0.734) known among double mode RR Lyrae stars. The period of V13 has been decreasing dramatically: it was $P_0 = 0.4830311$ d in the 60s (calculated from the data of Szeidl 1965) and it is now $P_0 = 0.4795043$ d. Both facts are alluding to strong evolutionary effects. V13 is maybe a ‘transient’ double-mode pulsator and not a ‘stable’ one (for definition of terms see Szabó et al. 2004).

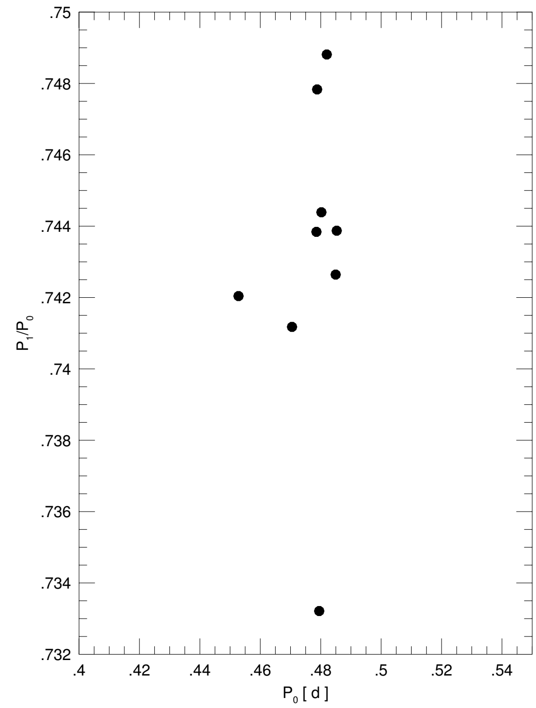

Figure 13. Petersen diagram of double-mode RR Lyrae in M3.

3.5.3 *Sudden changes in modal contents*

Concerning its double mode pulsators M3 seems to be unusual. The cluster is unique among the galactic globular clusters in that it has RRd variable stars with a dominant fundamental mode: V13, V166 and V251. Only very few RRd variables are presently known to have a dominant fundamental mode, and they all are field variables: AQ Leo, NSVS 5222076 (Oaster, Smith & Kinemuchi 2006) and some stars in the LMC (Alcock et al. 2000).

M3 is so far the only cluster in which RRd stars have been observed to switch from one dominant mode to another while remaining RRd type variables. Variations in the double-mode behaviour were observed in a number of RRd variable stars (Jerzykiewicz, Schult & Wenzel 1982, Jurcsik & Barlai 1990, Clement, Ferance & Simon 1993, 1997, Purdue et al. 1995). These variations concern initiation or cessation of the double-mode behaviour and changes in the length and amplitude ratios of the two periodicities. None of these studies reports a switch between dominant modes. However, for five RRd stars (V68, V99, V166, V200, V251) of the known nine in the globular cluster M3 such a possible mode switch was published by different authors (Corwin et al. 1999, Benkő & Jurcsik 2000, CC04). The case of V79 is different. As Clement et al. (1997) revealed this star switched from a single mode first overtone pulsation to double mode one.

It has to be stressed, that except for V79, the alleged mode switching was found within 1-2 years long time series. The ques-

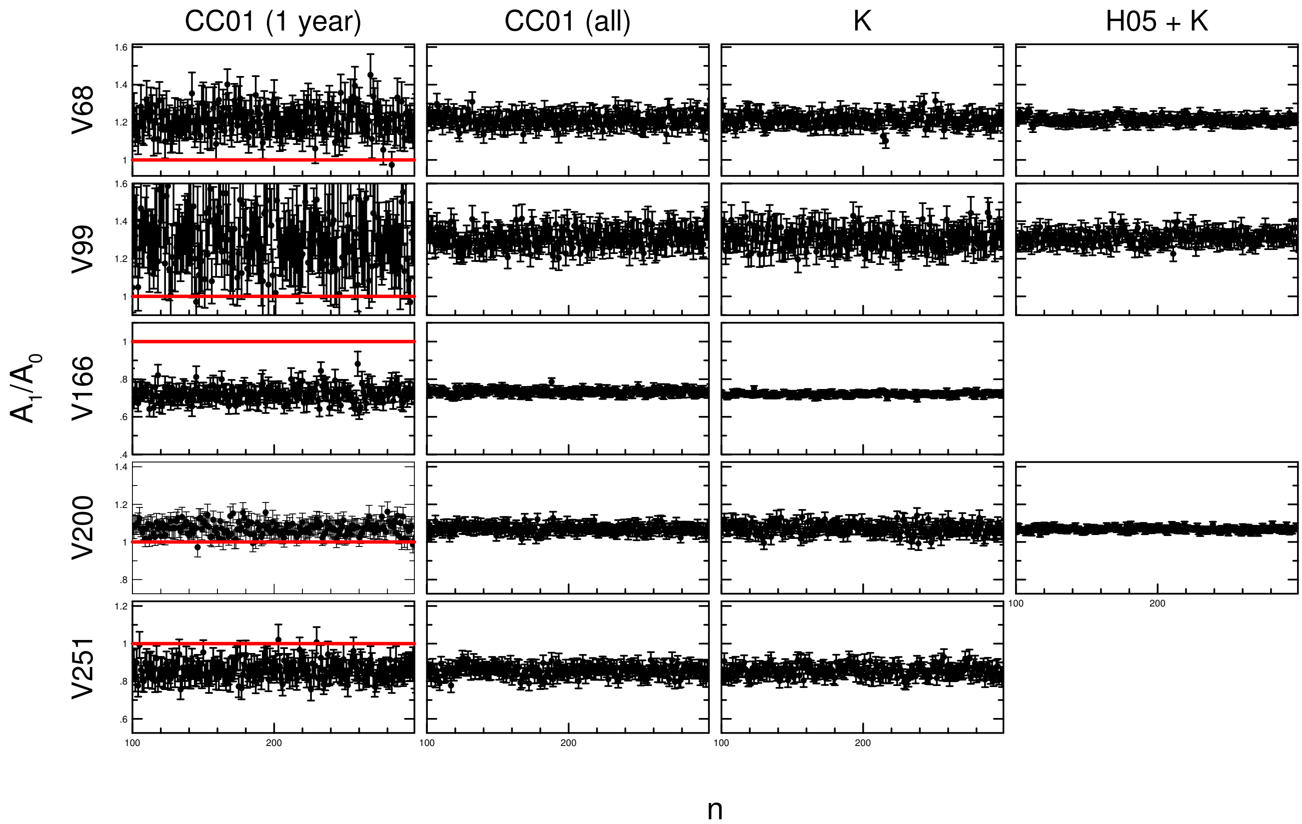

Figure 14. Calculated amplitude ratios (A_1/A_0) of synthetic light curves versus iteration number n. Here n are defined by $T_0 = t_0 + n * 0.01$ days; t_0 and T_0 are the epoch of the beginning of the synthetic light curves and a given sampled ones, respectively. The rms error of the fit is also marked for each point. Data distributions were taken from CC01 = Corwin & Carney (2001), H05 = Hartman et al. (2005), K = present work.

tion is whether they were real effects or just artifacts of data distributions?

To answer the question the best of (longest, homogeneous) CCD data sets were chosen as *reference light curve* for each RRd stars and were Fourier analyzed. First the frequency of the dominant mode was determined, then, after prewhitening the data with this frequency and its low order (7-15) harmonics, the frequency of the other mode was searched. If the spectral window did not allow to make clear distinction of the true period because of severe aliasing, then that possible frequency was accepted which was the closest to the frequency of a given mode as determined from other observations.

Using the found frequencies of the fundamental and first overtone modes, their harmonics and linear combinations in each data set, least square fit solutions were calculated in order to determine the amplitudes. The fitting formula was

$$m(t) = a_{00} + \sum_{i,j} a_{ij} \sin[2\pi\nu_{ij}(t - t_0) + \varphi_{ij}], \qquad (3)$$

where the designations are the usual: the Fourier amplitudes, a_{ij}, phases, φ_{ij}, the epocha, t_0, the frequencies, $\nu_{ij} = if_0 + jf_1$, i and j integers. As the parameters of the fits proved to be very sensitive to the number of harmonics and linear combinations concerned, we used only those components which actually appeared in the Fourier spectrum.

The modal content of the pulsation is quantified by the amplitude ratio A_1/A_0: Fourier amplitude of the first overtone mode $A_1(= a_{01})$ divided by the amplitude of fundamental mode $A_0(= a_{10})$. We would like to emphasize that the amplitude ratio defined above may differ significantly from the ratio of the total amplitudes which is generally used in the literature (e.g. Nemec & Clement 1989, Corwin et al. 1999).

The estimation of errors of amplitude ratios yields the key to justifying their possible changes. The systematic errors of the amplitude ratios were calculated by a simulation: synthetic time series were generated using the actual frequencies, amplitudes, phases of reference light curves with observational times of data sets from which mode switching were published. Fourier parameters were determined in the same way as for the reference data in 1000 simulations/each data set/each star. The initial epoch of simulated light curves (T_0) was shifted at each iteration steps with 0.01 days and white noise was also added with variance corresponding to the observed data. The mean results of these tests are plotted in Fig. 14.

- The errors of the amplitude ratios could be an order of magnitude higher than their fitting errors.
- The same sampling rate yields strongly different accuracy of amplitude ratio from star by star. Therefore, improving of time coverage of data reduces the systematic error but not equally.
- The amplitude ratios were severely affected by the value of the initial phase in those cases when only few nights of observations were involved even in high precision CCD observations.

These results suggest that the formerly published mode switching events were over-interpretations of badly sampled observations.

This conclusion are also supported by the theoretical calcu-

Table 8. The colour dependence of the amplitude ratios of multicolour CCD observations. $(A_1/A_0)_B$, $(A_1/A_0)_V$, $(A_1/A_0)_I$ denote the measured amplitude ratios in Johnson–Cousins *BVI* colours. Column Ref. show the sources of used data published by others.

Star	$(A_1/A_0)_B$	$(A_1/A_0)_V$	$(A_1/A_0)_I$	Ref.
V68	1.21 ± 0.06	1.21 ± 0.05	1.16 ± 0.23	1,3
V68	1.25 ± 0.06	1.23 ± 0.03	1.14 ± 0.05	2,4,5
V79	1.36 ± 0.12	1.37 ± 0.12		3
V79	1.72 ± 0.06	1.87 ± 0.05	1.80 ± 0.05	2,4,5
V87	2.45 ± 0.18	2.52 ± 0.25		1,3
V87	2.62 ± 0.15	2.28 ± 0.05	2.25 ± 0.08	4,5
V99	1.88 ± 0.07	1.48 ± 0.02		3,4,5
V166	0.72 ± 0.18	0.78 ± 0.09	0.59 ± 0.25	3
V166	0.65 ± 0.15	0.60 ± 0.05	0.62 ± 0.05	5
V200	1.11 ± 0.08	1.01 ± 0.07	1.13 ± 0.12	5
V251	0.49 ± 0.15	0.81 ± 0.10		5
V252	1.14 ± 0.05	1.10 ± 0.03		4,5

Ref.: 1. Carretta et al. (1998), 2. Kaluzny et al. (1998), 3. Corwin & Carney (2001), 4. Hartman et al. (2005), 5. this study.

lations of Buchler & Kolláth (2002) who showed that the mode-switching time scale is about hundred years in the most dramatic case.

3.5.4 Long time-scale stability of modes

To investigate the mode switching phenomenon in longer time-scale all published photometry (both photographic and CCD) was collected. Six RRd stars (V13, V68, V79, V87, V99, V166) have sufficient amount of historical data to study the possible long term changes of modal content. In the case of V13 and V79 no alternate periods have been found from any parts of historical data while due to their central position V200, V251 and V252 have not any old observations at all. For a practical review of published measurements of the remaining four stars see the lower panels of Fig. 15 where all different data sets are plotted denoting by different symbols.

The data were divided into groups containing as many measurements as possible in a moderate lengths of time interval where the periods seem to be constants. When different photometric observations belonged to the same group, zero point corrections were applied to match the photometries.

The effect of colour inhomogeneities on the amplitude ratios were tested by determining them using the different colours of all the available multicolour observations. The results are shown in Table 8. It can be seen, that the colour dependence of the amplitude ratio is generally smaller than the other errors entering in the results.

Each data group were analyzed and fitted in the same manner as it was described in Sec. 3.5.3. The only difference is the number of used Fourier harmonics in the case of photographic measurements (2-3).

In the top panels of the Figure 15 it can be seen that all studied RRd stars have a well defined amplitude ratio within the fitting error. Neither mode switching nor severe amplitude ratio modification were found.

3.6 Notes on individual stars

V4 Two RRab stars are separated by less than $0\farcs5$ (Guhathakurta et al. 1994) with periods of 0.585029 and 0.593069 days, respectively.

V5 The only star for which period of Blazhko cycle was estimated previously. The value of 194.551 days given by Panov (1980) seems to be not valid for CCD data. Unfortunately, determining correct period was unsuccessful.

V10 As opposed to C98 we can confirm light curve variations reported by Szeidl (1965). The star has a clearly visible Blazhko type amplitude modulation.

V18 Although, by earlier catalogues this star was classified as a Blazhko type RR Lyrae we could not detect such a behaviour on the basis of all available data sets (K98, CC01, H05 and present work).

V22 The light curve variations are certain from both available CCD photometries (CC01 and this work).

V23 Its brightness in band *I* is too high.

V29 The star is close to the V155. The period given by C98 is an alias. CC01 interchanged V155 with V29. V29 is a short period RRab with strong phase modulation.

V54 We agree with Szeidl (1965) that the star shows the Blazhko effect clearly, albeit by newer sources (e.g. Clement et al. 2001, CC01) this fact was neglected.

V44 CC04 reclassified it as an RRc variable star, but we agree with the earlier work CC01 and the classification type as RRab with Blazhko modulation.

V70 The star lies definitely above the horizontal branch and it has an RRc like light curve with an unusually long period. It might be an evolved RR Lyrae stars like V129, V170 and V261. We cannot find a common period with any of the C98, K98 nor CC01. This indicates strong period changes.

V77 Unusually short period RRab type star. As it was noted by CC01 its colour index $B-V$ placed them slightly to the RRc side of the CMD.

V95 This variable star was reported by the ROTSE I survey as a new one by Akerlof et al. (2000) as ROTSE1 J134159.98+282257.0.

V104 Combining CCD data (K98, CC01 and ours) on this star its Blazhko behaviour is obvious.

V114 Very few points. The star is lying far from the centre and out of most of our frames.

V122 See comments at V229.

V129 Similar to V70.

V140 It has a light curve modulation both in phase and amplitude as it was already mentioned by Szeidl (1965). Most probably this star is an RRc with Blazhko effect.

V146 C98, CC01 and CC04 have measured the V222 instead of this star.

V149 The light curve variability is evident both from ours and from CC01's data.

V155 CC01 have interchanged this star with V29.

V168 In addition to V140 this star is another example for the Blazhko type RRc variables in the cluster.

V170 It is a variable star like V70. Unfortunately, it is merging with V136.

V172 We have found the period to be much shorter (~ 0.543 d) than it was given by The Catalogue (0.594 d) unless they accidentally reversed the last two digits. A possible phase modulation was detected as well.

V178 The pulsation frequency of the nearby V152 can be recognized clearly in the Fourier spectrum of V178. Modulation of V178 is probably caused by this neighbourhood.

V191 We reported here its evident Blazhko behaviour for the first time.

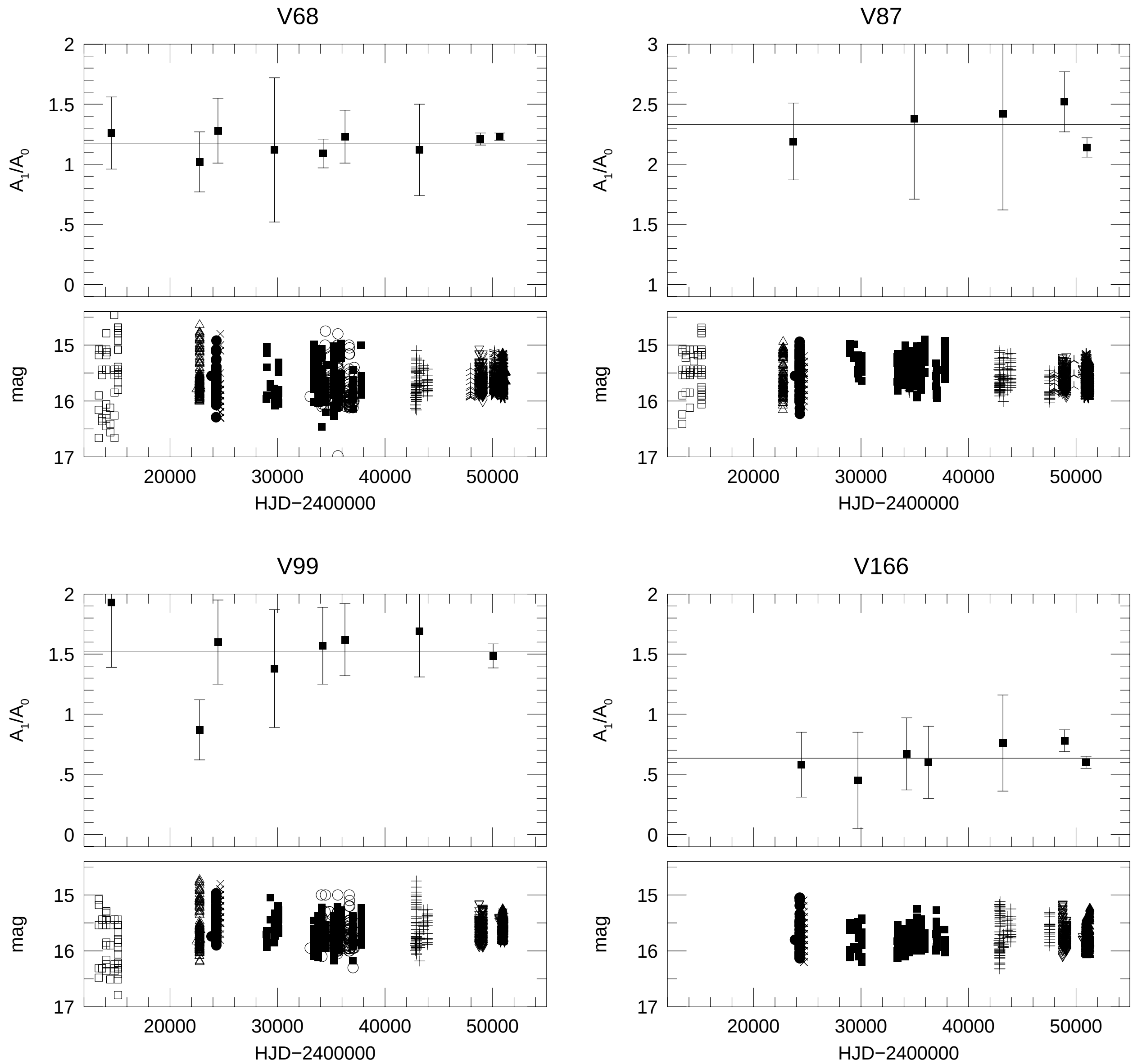

Figure 15. Upper panels: amplitude ratios of the RRd variables in M3 determined for the different group of the data. The error-bars denote the rms error of Fourier fits. Lower panels: the time distributions of the photometric observations of each variables. Symbols: open squares: Bailey (1913), open triangles: Larink (1922), dots: Müller (1933), crosses: Greenstein (1935), filled squares: Szeidl (1965, 1973), open circles: Kukarkin & Kukarkina (1961, 1970), letters Y: Roberts & Sandage (1955), pluses: Konkoly Observatory (unpublished), trident stars: Carretta et al. (1998), pentagrams: Kaluzny et al. (1998), open triangles in upside down: Corwin & Carney (2001), stars: Hartman et al. (2005), filled triangles: present work.

V207 S02 published a period 0.4291 d. We agree with CC01 in a much shorter one: 0.345307 d.

V211 The Blazhko effect noted in Table 4 is clearly seen in its phase diagram.

V218 shows Blazhko effect with large amplitude modulation.

V221 S02 have suggested its period as 0.6098 d. Folding our data with the period 0.3787 d given by CC01 produced a slightly smaller scatter. The colour indices are those of type RRc stars.

V222 CC01 and CC04 have interchanged this star with V146.

V229 S02 obtained period 0.6877 d. We have found a much shorter one: 0.49917 d.

V234 CC01 measured this star under the name of V164 as it was corrected by CC04. S02 found a very different period: 0.5495 d.

V239 CC01 published a period as 0.333431 d. CC04 corrected it to 0.66949 d and S02 found a similar one (0.6766 d). Nevertheless this period is an alias! Preparing the phase diagram to this period two maxima and minima are seen within one cycle. Period 0.503982 d from our data yields correct phase diagrams for all data sets.

V242 Period of S02 0.6513 d is significantly different from the common period of CC04 and ours.

V250 As it was shown in Section 3.2, actually it is two overlapping variable stars.

V259 was found by S02 as an RRab star with the period of about a half a day. Our data fit is better with the RRc period of CC01 and CC04.

V261 is also an evolved star like V70, V129 and V170.

V266 was not included in any of former time series studies. The period determined from our data suggest an RRc type star. The shape of the folded light curve shows also a normal first overtone mode pulsating RR Lyrae.

V270 It is two blended variable stars. See Section 3.2 for details.

4 SUMMARY

We have conducted the most extensive CCD *BVI* survey of variables in the globular cluster M3 covering 238 stars. Image subtraction photometric data transformed into the Johnson-Cousins magnitude scale, light curves, periods and average magnitudes were presented

- *Varibility search* In addition to our new variable stars published in Bakos et al. (2000), we have discovered three further RR Lyrae stars. We also found four non-variable stars in the instability strip. Period, brightness were improved for most of the stars and several confused identifications were clarified.
- *Color – magnitude diagrams* At least three significant subgroups have been separated by hierarchical clustering method in the CMDs of the RRab stars. The identified three clusters agree well with previous studies of Jurcsik et al. (2003) and with CC05 when we contract the appropriate clusters to 2 groups from our three ones or 3 from the four subgroups separated by Jurcsik et al. (2003).
- *Period – brightness relations* A clear correlation have been found between period of RRab stars and their $\langle I \rangle$ magnitudes. The relation is in good agreement with synthetic horizontal branch calculation published by Catelan et al. (2004).
- *Modulated RR Lyrae stars* We identified 66 modulated (Blazhko) RR Lyrae stars in our sample where five of them are new discoveries. For six stars the ambiguous Blazhko classification have been verified. Two modulated RRc variable stars were found in the cluster. By combining the available CCD data modulation periods were estimated for 13 Blazhko star for the first time.
- *Double mode RR Lyraes* Investigating the Fourier spectra of V252 its double-mode pulsation has been confirmed. Analyzing all available photometric data on RRd stars of the cluster no undoubted modal content change, neither on shorter nor longer time scales have been detected except for the case of V79. It was shown, that to detect a mode-switching it is practically impossible from available CCD photometric materials containing observing runs from 1-3 seasons.

At the end of this paper we would like to call the attention to the necessity of further long CCD time series observation of the variable stars of M3. To answer the question of modal changes of the double-mode variable stars or to find correct period for all modulated stars and many similar problems need long-term, homogeneous and high precision photometry.

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Table 9: The basic parameters of the variables in M3.

ID	Period	Type	$\overline{B}$	$\langle B\rangle$	$\overline{V}$	$\langle V\rangle$	$\overline{I}$	$\langle I\rangle$	$\mathrm{Ref}_{B,V,I}$	Comm.
1	0.5205963	RRab	16.004	15.900	15.688	15.631	15.248	15.227	R R R	3
3	0.5581979	RRab	15.939	15.857	15.661	15.612	15.175	15.155	R R R	Bl,3
4n	0.585029	RRab							R R R	c
4s	0.593069	RRab							R R R	c
5	0.505703	RRab	15.986	15.974	15.725	15.714	15.243:	15.238:	A A S	Bl,c
6	0.5143327	RRab	16.083	16.003	15.755	15.703	15.294	15.276	R R R	1
7	0.4974248	RRab	15.980	15.889	15.694	15.638	15.248	15.230	R R R	Bl,2
8	0.636728	RRab	15.808	15.787	15.649	15.634	14.902	14.898	R: R: R:	3
9	0.5415553	RRab	16.031	15.957	15.689	15.645	15.233	15.217	A A S	2
10	0.5695465	RRab	16.057	16.004	15.684	15.647	15.162	15.152	S S S	Bl,2,c
11	0.5078915	RRab	15.993	15.873	15.706	15.636	15.188	15.166	S S S	3
12	0.3179347	RRc	15.840	15.814	15.621	15.606	15.281	15.275	R R R	
13	0.4795043	RRd	15.931	15.917	15.691	15.684	15.243	15.239	R R R	$P_1 = 0.351579$
14	0.6359002	RRab	15.914	15.867	15.581	15.551	15.059	15.050	R R R	Bl,4
15	0.5300874	RRab	15.919	15.833	15.667	15.616	15.159	15.144	A A A	3
16	0.5114943	RRab	16.110	16.020	15.759	15.701	15.307	15.288	A A S	1
17	0.5761594	RRab	16.060	16.020	15.694	15.680	15.136	15.127	A A A	Bl
18	0.5164527	RRab	16.179:	16.068:	15.751	15.698	15.251:	15.231:	A A A	c,1
19	0.6319774	RRab	16.109	16.093	15.689	15.681	15.148	15.145	A A S	2
20	0.4912636	RRab	16.075	15.988	15.707	15.656	15.233	15.215	S A S	Bl
21	0.515758	RRab	16.103	16.012	15.742	15.690			S S S	a,2
22	0.4814205	RRab	16.085:	15.989:	15.762	15.698	15.292	15.269	S A S	Bl,c
23	0.5953834	RRab	15.934	15.884	15.679	15.639	14.683	14.677	S: S: S:	Bl,c
24	0.663380	RRab	15.961	15.914	15.558	15.533	15.068	15.060	A S S	Bl,4
25	0.4800623	RRab	16.002	15.886	15.747	15.681	15.294	15.271	R R R	2
26	0.5977427	RRab	15.962	15.891	15.639	15.596	15.002	14.991	A S S	3
27	0.5790682	RRab	16.031	15.972	15.679	15.649	15.242	15.230	R R R	2
28	0.470617	RRab	15.982	15.911	15.709	15.655	15.281	15.268	R R R	Bl
29	0.4717879	RRab	15.918	15.874	15.821	15.769	15.271	15.265	R R R	Bl,m,c
30	0.5120888	RRab	15.993	15.895	15.726	15.676	15.265	15.248	R R R	m,2
31	0.5807219	RRab	15.946	15.829	15.612	15.556	15.160	15.140	R R R	4
32	0.4953506	RRab	16.043	15.924	15.736	15.675	15.265	15.244	R R R	2
33	0.5252299	RRab	15.944	15.877	15.705	15.670	15.200	15.191	R R R	Bl
34	0.5591031	RRab	16.089	16.067	15.787	15.768	15.179	15.177	A A S	Bl
35	0.5305483	RRab	15.998	15.878	15.695	15.620	15.132:	15.112:	A A S	Bl
36	0.5455929	RRab	15.944	15.854	15.660	15.601	15.235	15.214	S A S	3
37	0.3266378	RRc	15.928	15.903	15.693	15.679	15.307	15.302	S A S	
38	0.5580110	RRab	16.108	16.086	15.765	15.701	15.191	15.171	A A S	Bl
39	0.5870758	RRab	16.093	16.021	15.701	15.668	15.220	15.206	A A S	Bl
40	0.5515394	RRab	16.110	16.038	15.711	15.672	15.231	15.218	A A S	2
41	0.4858777	RRab	16.115	16.001	15.712	15.659	15.311	15.292	R R R	Bl
42	0.5900938	RRab	15.854	15.724	15.563	15.497	15.103	15.079	R R R	4
43	0.5405129	RRab	16.089	16.006	15.719	15.671	15.244	15.228	R R R	Bl,1
44	0.5063801	RRab	15.972	15.912	15.659	15.615	15.171	15.160	A A S	Bl
45	0.5369002	RRab	16.118	16.056	15.706	15.675	15.259	15.247	S A S	Bl
46	0.6133832	RRab	16.050	16.028	15.716	15.703	15.158	15.153	R R R	1
47	0.5409128	RRab	15.995	15.941	15.673	15.648	15.200	15.190	R R R	Bl
48	0.6278299	RRab	15.952	15.922	15.631	15.615			S S S	3
49	0.5482048	RRab			15.696	15.645			S A S	Bl
50	0.5128392	RRab	16.038	15.991	15.674	15.645	15.173	15.165	S A S	Bl
51	0.5839702	RRab	16.093	16.041	15.688	15.658	15.194:	15.187:	S S A	2
52	0.5162299	RRab			15.773	15.756			R R R	Bl
53	0.5048789	RRab	16.052	15.939	15.731	15.669	15.301	15.281	R R R	2
54	0.5062570	RRab	16.102	16.044	15.753	15.714	15.356	15.344	R R R	Bl,c
55	0.5298207	RRab	16.063	15.976	15.724	15.673			A A S	2
56	0.3296000	RRc	15.885	15.866	15.658	15.642	15.257	15.253	A A S	
57	0.5121885	RRab	16.078	16.001	15.748	15.699	15.378	15.357	S S S	2
58	0.5170549	RRab	15.936	15.825	15.670	15.600	15.218	15.196	R R R	3

Table 9: (continued)

ID	Period	Type	$\overline{B}$	$\langle B\rangle$	$\overline{V}$	$\langle V\rangle$	$\overline{I}$	$\langle I\rangle$	$\mathrm{Ref}_{B,V,I}$	Comm.
59	0.5888236	RRab	16.130	16.071	15.677	15.649	15.179	15.170	A A S	Bl,2
60	0.7077280	RRab	15.970	15.936	15.525	15.506			S A S	4
61	0.5209053	RRab	16.062:	15.991:	15.733	15.690	15.220:	15.207:	S A S	Bl
62	0.6524099	RRab	16.055	16.027	15.657	15.643	15.091	15.085	A S S	Bl,3
63	0.5704004	RRab	16.081	16.052	15.710	15.670	15.170	15.163	S A S	Bl
64	0.6054588	RRab	16.080	16.045	15.693	15.672	15.162:	15.156:	A S S	2
65	0.6683474	RRab	15.940	15.875	15.531	15.497	15.004	14.991	A S A	4
66	0.6201807	RRab	16.026	15.991	15.659	15.641	15.150	15.142	R R R	Bl
67	0.5683324	RRab	16.058	15.976	15.683	15.635	15.218	15.200	R R R	Bl
68	0.3559911	RRd	15.929	15.916	15.636	15.632	15.267	15.265	R R R	$P_0 = 0.478585$
69	0.5666159	RRab	16.101	16.037	15.704	15.672	15.219	15.208	R R R	2
70	0.4863518	RRc(?)	15.684	15.670	15.392	15.386	14.956	14.953	R R R	c
71	0.5490530	RRab	16.048	16.008	15.825	15.796	15.033	15.024	A: S: A:	Bl?,m
72	0.4560780	RRab	16.076	15.953	15.743	15.670	15.349	15.320	A A S	2
73	0.673611	RRab	16.054	16.047	15.629	15.626	15.093	15.092	S A S	3
74	0.4921525	RRab	16.129	15.994	15.759	15.692	15.325	15.304	R R R	2
75	0.3140767	RRc	15.914	15.890	15.640	15.626	15.325	15.319	R R R	
76	0.5017627	RRab	16.076	15.967	15.778	15.720	15.317:	15.294:	R R R	1
77	0.4593499	RRab	15.980	15.870	15.816	15.743	15.333	15.311	R R R	Bl,c,1
78	0.61192	RRab	15.969	15.944	15.613	15.602	15.117	15.110	R R R	Bl
79	0.3580727	RRd	15.999	15.988	15.749	15.742	15.268	15.266	A A S	$P_0 = 0.478814$
80	0.5383788	RRab	16.027	15.955	15.679	15.632	15.218	15.205	A A S	Bl
81	0.5291220	RRab	16.078	15.986	15.730	15.679	15.277:	15.255:	A A S	2
83	0.5012631	RRab	16.057:	15.960:	15.733	15.674	15.248:	15.228:	A A A	2
84	0.5957289	RRab	16.082	16.042	15.678	15.658	15.166	15.158	R R R	2
85	0.3558191	RRc	15.780	15.754	15.591	15.574	15.180	15.176	A S S	
86	0.2926596	RRc	15.937	15.909	15.691	15.673	15.381	15.375	S A S	
87	0.3574779	RRd	15.870	15.856	15.580	15.571	15.190	15.187	R R R	$P_0 = 0.480229$
88	0.2987499	RRc	15.900	15.868	15.706	15.687	15.183	15.179	R R R	
89	0.5484803	RRab	16.040	15.955	15.689	15.647	15.190	15.176	R R R	2
90	0.5170308	RRab	16.111:	16.008:	15.727	15.673	15.207:	15.190:	S A S	2
92	0.5035471	RRab	16.041	15.949	15.723	15.672	15.222	15.201	A A S	a,2
93	0.6022960	RRab	16.060	16.021	15.662	15.640	15.151	15.143	A A S	2
94	0.5236940	RRab	16.082	15.993	15.725	15.671	15.252	15.235	A A S	2
95	103.5	long							A A A	c
96	0.4994150	RRab	16.023	15.918	15.719	15.652			A S S	a,2
97	0.3349326	RRc	15.947	15.933	15.712	15.703	15.359	15.355	S S S	
99	0.3609319	RRd	15.832:	15.824:	15.592	15.590	15.098:	15.096:	S A S	$P_0 = 0.482006$
100	0.6188126	RRab	16.074	16.045	15.721	15.705	15.171	15.165	R R R	1
101	0.6438879	RRab	16.117	16.093	15.703	15.692	15.169	15.166	R R R	Bl,2
104	0.5699259	RRab	15.883	15.773	15.592	15.536	15.164	15.145	R R R	Bl,c,4
105	0.2877440	RRc	15.802	15.791	15.586	15.580	15.308	15.306	R R R	
106	0.5471316	RRab	16.112	16.039	15.712	15.675	15.237	15.223	R R R	Bl
107	0.3090378	RRc	15.917	15.890	15.690	15.674	15.299	15.293	A A S	
108	0.519610	RRab	16.093	16.000	15.762	15.708	15.260	15.241	S S S	a,1
109	0.5339205	RRab	16.030	15.945	15.746	15.695	15.227	15.211	R R R	1
110	0.5354602	RRab	16.035	15.959	15.747	15.698	15.222	15.206	R R R	Bl
111	0.5101919	RRab	16.007	15.938	15.754	15.694	15.314	15.290	R R R	Bl
114	0.597723	RRab							A A A	c
116	0.514811	RRab	16.116	16.026	15.762	15.706			A A S	a,1
117	0.6005350	RRab	15.986	15.913	15.642	15.592	15.059	15.042	A A A	Bl
118	0.4993905	RRab	16.092	15.987	15.730	15.672			A A S	a,2
119	0.517690	RRab	16.062:	15.966:	15.729	15.670	15.201:	15.182:	A A S	2
120	0.640140	RRab	16.023	16.007	15.637	15.628	15.093	15.090	A A S	3
121	0.5352076	RRab	16.050	16.016	15.736	15.704	15.292	15.286	R R R	Bl
122	0.5159519	RRab							R R R	m,c
124	0.752439	RRab	16.012	16.000	15.550	15.544	14.944	14.942	S S S	4
125	0.3498227	RRc	15.978	15.959	15.740	15.730	15.224	15.220	S S S	

Table 9: (continued)

ID	Period	Type	$\overline{B}$	$\langle B\rangle$	$\overline{V}$	$\langle V\rangle$	$\overline{I}$	$\langle I\rangle$	Ref$_{B,V,I}$	Comm.
126	0.3484062	RRc	15.903	15.887	15.631	15.622	15.275	15.271	R R R	
128	0.2920410	RRc	15.864	15.840	15.653	15.637	15.350	15.344	R R R	
129	0.4060852	RRc(?)	15.778	15.760	15.500	15.492	15.073	15.070	R R R	c
130	0.5660594	RRab	15.972	15.958	15.679	15.671	15.160	15.156	R R R	Bl
131	0.2976890	RRc	15.904	15.878	15.695	15.679	15.354	15.349	R R R	
132	0.3398560	RRc	15.912	15.895	15.632	15.622	15.251	15.249	R R R	
133	0.5507236	RRab	16.048	15.977	15.734	15.701	15.275	15.261	R R R	Bl,2
134	0.6180597	RRab	15.995	15.971	15.680	15.663	15.145	15.139	R R R	2
135	0.5683920	RRab	16.006	15.965	15.743	15.706	15.239	15.228	R R R	m
136	0.617179	RRab	15.986	15.957	15.632	15.616	15.167	15.161	R R R	m,3
137	0.5751482	RRab	16.001	15.950	15.651	15.619	15.208	15.196	R R R	3
138	53.28	long							A A A	
139	0.560000	RRab	15.961	15.841	15.648	15.587	15.184	15.160	R R R	4
140	0.3331388	RRc	15.740	15.715	15.542	15.530	15.228	15.225	R R R	Bl,c
142	0.5686274	RRab	16.100	16.029	15.713	15.679	15.279	15.266	R R R	2
143	0.5965350	RRab	15.749	15.662	15.437	15.396	15.010	14.993	R R R	Bl
144	0.5967843	RRab	16.042	16.000	15.698	15.675	15.144	15.136	R R R	2
145	0.5144880	RRab	15.825	15.752	15.539	15.499	14.972	14.960	R R R	m
146	0.5021930	RRab	16.006	15.905	15.734	15.674	15.266	15.246	R R R	c
147	0.3464809	RRc	15.964	15.943	15.733	15.722	15.304	15.300	R R R	
148	0.4672693	RRab	16.173	16.026	15.823	15.752	15.366	15.343	R R R	m
149	0.5481796	RRab	16.083	15.992	15.679	15.641	15.277	15.250	R R R	Bl,c
150	0.5239248	RRab	16.066	15.978	15.737	15.687	15.256	15.238	R R R	Bl
151	0.5168247	RRab	15.991:	15.928:	15.647	15.621	15.273	15.260	R R R	Bl
152	0.3261308	RRc	15.823	15.805	15.551	15.540	15.294	15.289	R R R	
154	15.2842	WVir							R R R	
155	0.338050	RRc	15.815	15.805	15.667	15.660	15.257	15.254	R R R	m,c
156	0.531986	RRab	16.015	15.919	15.726	15.681	15.302	15.282	R: R R	2
157	0.542851	RRab	16.026	15.985	15.750	15.728	15.295	15.288	R R R	Bl
159	0.533890	RRab	15.733	15.678	15.407	15.380	14.914	14.903	R R R	m
160	0.657330	RRab	15.879	15.82	15.559	15.538	15.096	15.086	R R R	Bl,4
161	0.526495	RRab	16.076	15.966	15.754	15.693	15.245	15.221	R R R	Bl
165	0.4836315	RRab	15.920	15.802	15.647	15.585	15.244	15.222	R R R	
166	0.4849529	RRd	15.986	15.969	15.809	15.797	15.275	15.271	R R R	$P_1 = 0.360147$
167	0.6439710	RRab	16.074	16.064	15.684	15.677	15.133	15.131	R R R	1
168	0.2759408	RRc	15.786	15.772	15.628	15.617	15.278	15.274	R R R	Bl,c
170	0.4323323	RRc(?)	15.613	15.590	15.416	15.406	15.044	15.039	R R R	m,c
171	0.303300	RRc	15.842	15.811	15.631	15.613	15.432	15.427	R R R	m
172	0.5422899	RRab	16.074	15.995	15.788	15.741	15.291	15.276	R R R	c,1
173	0.6070027	RRab	15.974	15.901	15.659	15.622	15.135	15.123	R R R	3
174	0.5913187	RRab	16.019	15.988	15.738	15.722	15.370	15.361	R R R	
175	0.569697	RRab	16.092	16.026	15.699	15.665	15.183	15.169	R R R	2
176	0.5396109	RRab	16.197	16.128	15.733	15.707	15.235	15.224	R R R	Bl
177	0.3483497	RRc	15.699	15.671	15.534	15.517	15.270	15.264	R R R	
178	0.2669625	RRc	15.905	15.891	15.705	15.699	15.480	15.478	R R R	m,c
180	0.6091004	RRab	16.050	16.017	15.702	15.679	15.347	15.336	R R R	Bl,2
181	0.6638463	RRab	15.893	15.870	15.540	15.528			R R	
184	0.5312073	RRab	15.971	15.948	15.690	15.660			R R	m
186	0.6634230	RRab	15.944	15.931	15.619	15.613	15.100	15.095	R R R	3
187	0.586257	RRab	16.164	16.094	15.731	15.709	15.280	15.269	R R R	Bl?,1
188	0.26652823	RRc	15.958	15.944	15.789	15.782	15.530	15.527	R R R	
189	0.61294	RRab	16.161	16.116	15.706	15.687	15.184	15.175	R R R	m,2
190	0.5227970	RRab	16.014	15.929	15.737	15.683	15.234	15.222	R R R	2
191	0.519206	RRab	15.996:	15.926:	15.736	15.702	15.333	15.321	R R R	Bl,c
193	0.747860	RRab	15.853	15.772	15.502	15.466	14.984	14.970	R: R: R:	m,4
194	0.4892000	RRab	15.910:	15.772:	15.679	15.628	15.270	15.255	R R R	
195	0.64408	RRab	15.971	15.953	15.612	15.606	15.172:	15.169:	R R R	3
197	0.4998971	RRab	16.158	16.016	15.788	15.719			R R R	a,1

Table 9: (continued)

ID	Period	Type	$\overline{B}$	$\langle B\rangle$	$\overline{V}$	$\langle V\rangle$	$\overline{I}$	$\langle I\rangle$	$\mathrm{Ref}_{B,V,I}$	Comm.
200	0.3610002	RRd	15.951	15.934	15.568	15.561	15.037	15.035	R R R	$P_0 = 0.485298$
201	0.54054	RRab	16.083	15.964	15.706	15.670	15.485	15.454	R R R	
202	0.773571	RRab	15.961	15.959	15.559	15.558	14.977:	14.977:	A A S	4
203	0.2897940	RRc	15.858	15.855	15.593	15.592	15.253	15.252	S A S	
207	0.345307	RRc	15.838	15.819	15.628	15.617	15.319	15.315	R R R	c
208	0.3383847	RRc	15.872	15.853	15.709	15.698	15.327	15.323	R R R	
209	0.3482784	RRc	15.801	15.780	15.621	15.607	15.251	15.246	R R R	
210	0.352947	RRc	15.837	15.820	15.683	15.672	15.230	15.223	R R R	
211	0.558205	RRab	15.988	15.948	15.781	15.760	15.236	15.227	R: R: R:	Bl,m,c
212	0.5421882	RRab	15.985	15.919	15.659	15.622	15.429:	15.411:	R R R	Bl
213	0.299955	RRc	15.700	15.677	15.513	15.501	15.154	15.151	R R R	
214	0.53952	RRab	15.994	15.911	15.741	15.706	15.323	15.305	R R R	1
215	0.528746	RRab	15.935:	15.885:	15.607	15.585	15.339:	15.324:	R R R	Bl
216	0.3464803	RRc	15.929	15.908	15.677	15.667	15.492	15.482	R R R	
218	0.544862	RRab	16.108	16.075	15.708	15.658	15.249	15.244	R R R	Bl,c
219	0.6136073	RRab	16.011	15.993	15.717	15.707			R R	
220	0.6001119	RRab	15.977	15.946	15.724	15.710	15.205	15.197	R R R	Bl?,1
221	0.3787879	RRc	15.540	15.520	15.512	15.499	15.290	15.283	R R R	m,c
222	0.5967446	RRab	15.986	15.931	15.598	15.572	15.050	15.042	R R R	c,3
223	0.3291883	RRc	15.840	15.811	15.589	15.576	15.289	15.284	R R R	
225	?	long							A A A	
226	0.4884239	RRab	16.029	15.934	15.736	15.685	15.419:	15.378:	R R R	m
229	0.4991777	RRab			15.736	15.682	15.308:	15.289:	R R R	m,c
234	0.508039	RRab	16.046	15.963	15.685	15.645	15.445	15.428	R R R	c
235	0.759846	RRab	15.882	15.860	15.539	15.526	14.966	14.961	R R R	4
236	?	long							A A A	
239	0.503982	RRab	15.907	15.815	15.754	15.700	15.282	15.258	R: R: R:	Bl,c,1
240	0.2760172	RRc	15.623	15.609	15.516	15.508	15.045	15.043	R: R: R:	
241	0.596172	RRab			15.494	15.441	15.347	15.315	R R	m
242	0.5964335	RRab	16.016	15.993	15.588	15.576			R R	m.c
243	0.634627	RRab	15.911	15.891	15.640	15.629	15.012	15.007	R R R	Bl?,3
244	0.5378472	RRab	15.947:	15.840:	15.728	15.682	15.327:	15.309:	R R R	1
245	0.2840324	RRc	15.886	15.853	15.704	15.689	15.346	15.338	R R R	
246	0.3391374	RRc	15.803	15.788	15.635	15.622	15.249	15.244	R R R	
247	0.6053555	RRab	16.084	16.05	15.688	15.670	15.182	15.174	R R R	2
248	0.5097854	RRab	16.099	16.042	15.613	15.603	15.260	15.248	R R R	m
249	0.5330032	RRab	15.942	15.879	15.688	15.640	15.135	15.123	R R R	3
250n	0.5671499	RRab			15.523	15.506			R	m,c
250s	0.5861391	RRab	15.847	15.743	15.537	15.490	15.383	15.344	R R R	m,c
251	0.4705028	RRd	15.75:	15.714:	15.608	15.596			R R	$P_1 = 0.348726$
252	0.3359896	RRd			15.728	15.719			R R	$P_0 = 0.452791$
253	0.3326257	RRc	15.561:	15.543:	15.504	15.493	15.182	15.176	R R R	
254	0.605656	RRab	15.794	15.772	15.545	15.531	15.200	15.193	R: R R	
255	0.5726891	RRab	16.071	16.004	15.600	15.581			R: R:	
256	0.3180587	RRc	15.912	15.894	15.704	15.692	15.418	15.412	R R R	
257	0.6019823	RRab	16.056	15.990	15.733	15.705	15.110:	15.089:	R R R	Bl?
258	0.713396	RRab	16.006	15.984	15.599	15.587	15.072	15.068	R R R	4
259	0.3335311	RRc	15.999	15.968	15.733	15.720	15.312	15.309	R R R	c
260	?	long							A A A	
261	0.4448913	RRc(?)	15.422	15.413	15.195	15.187	15.110	15.107	R: R: R:	c
263	0.2168508	SXPhe	17.443	17.437	17.046	17.043	18.009	17.991	R R R	
264	0.356466	RRc	15.622	15.599	15.474	15.465	15.143	15.139	R: R: R:	
266	0.3424518	RRc	15.849	15.834	15.516	15.508	15.308	15.307	R: R: R:	c
269	0.3592861	RRc	15.600	15.574	15.510	15.505	15.090	15.093	R: R: R:	
270n	0.625819	RRab							R R R	m,c
270s	0.690195	RRab							R R R	m,c
271	0.632794	RRab	16.031	15.997	15.640	15.624	15.182	15.175	R R R	3
272	?	long							A A A	

Table 9: (continued)

ID	Period	Type	$\overline{B}$	$\langle B\rangle$	$\overline{V}$	$\langle V\rangle$	$\overline{I}$	$\langle I\rangle$	$\mathrm{Ref}_{B,V,I}$	Comm.
273	46.43	long							A A A	
274	?	long							A A A	
N1	0.071628533	SXPhe	17.719	17.660	17.454	17.428	17.212	17.201	R R R	
N2	0.251010	RRc	15.707	15.700	15.535	15.534			R: R:	
N3	0.29654065	RRc	15.688	15.681	15.547	15.544	15.426	15.425	R: R: R:	

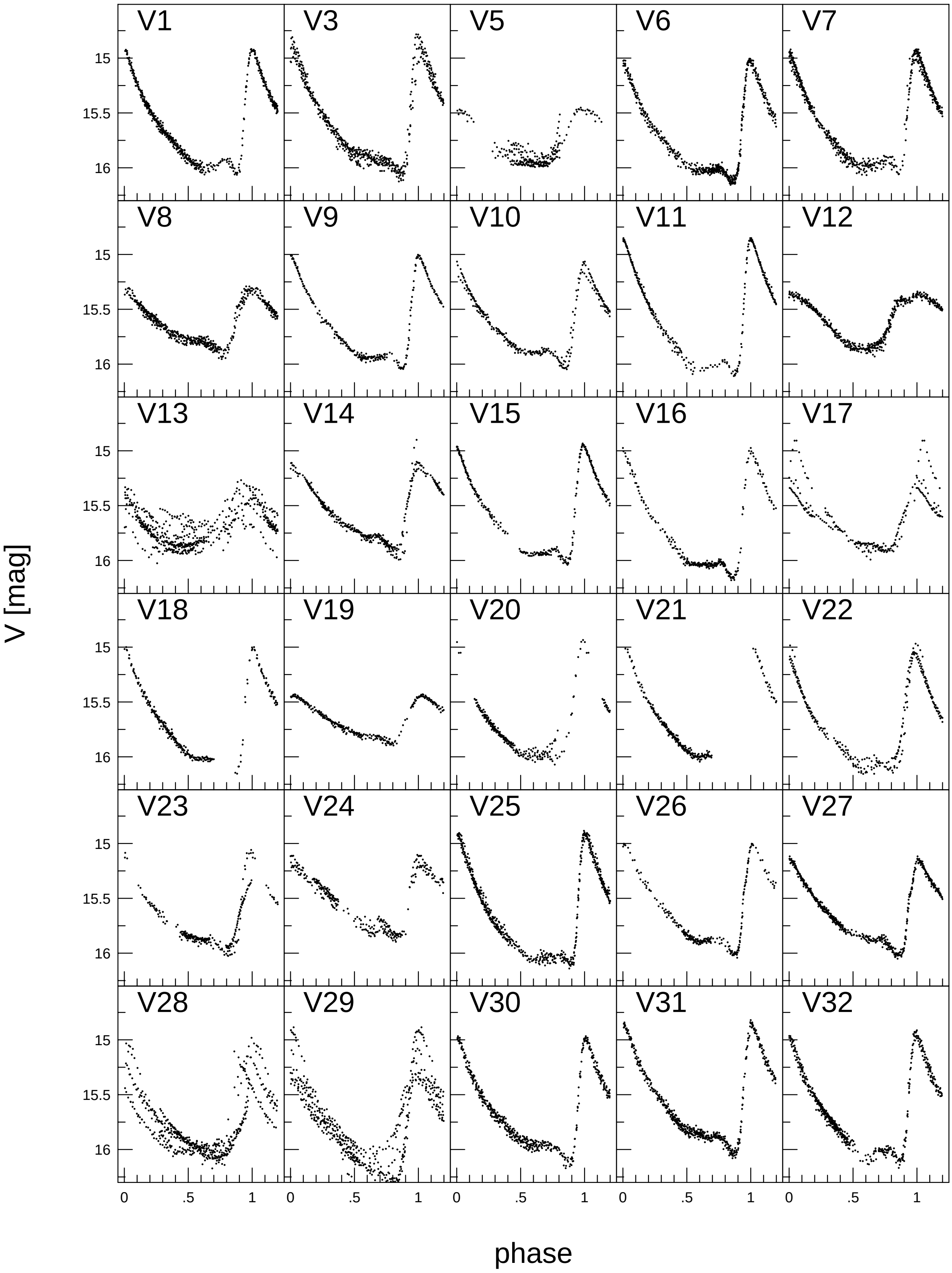


Figure 16. The V phase diagrams of the measured variables.

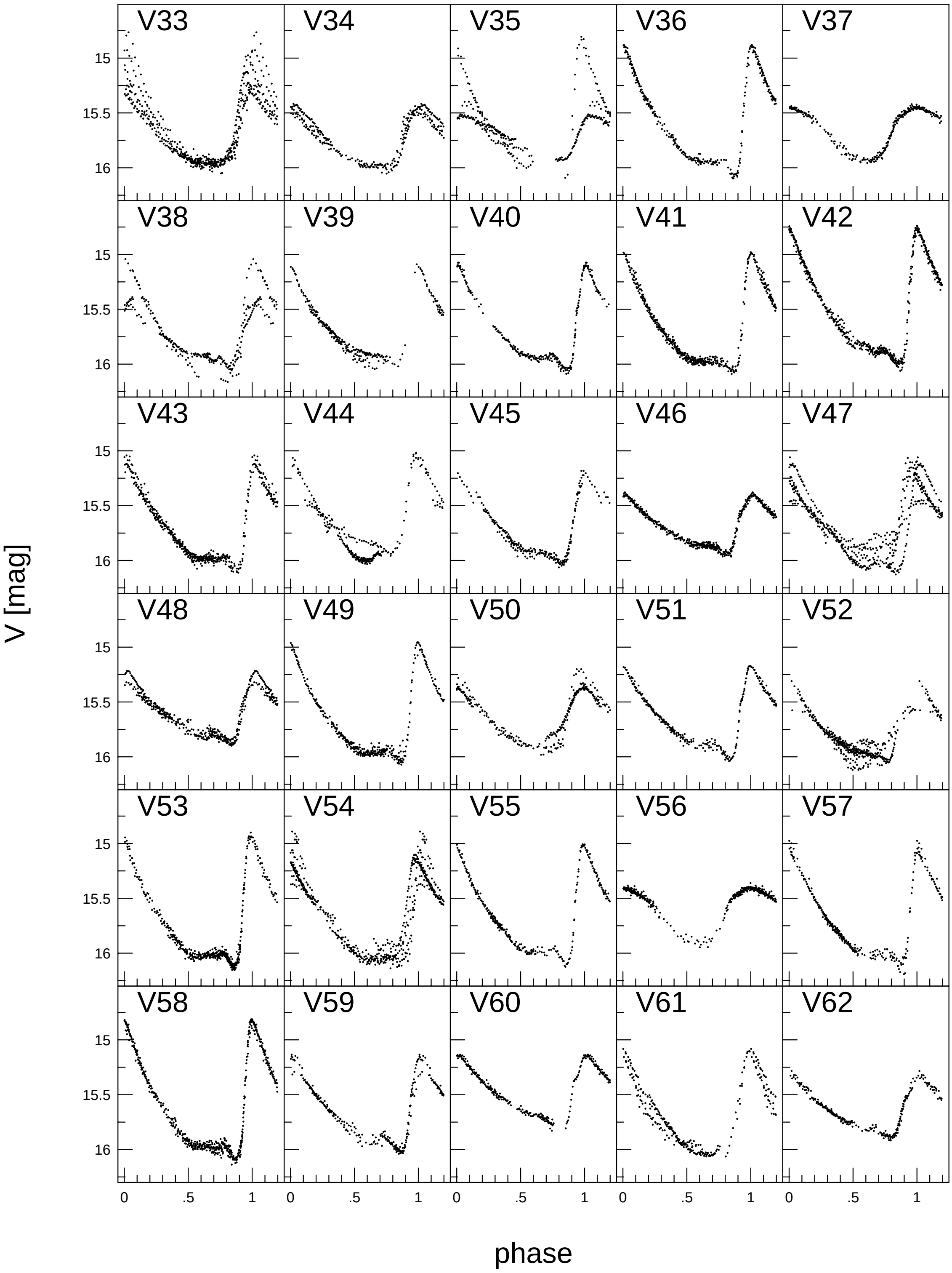


Figure 16. (continued)

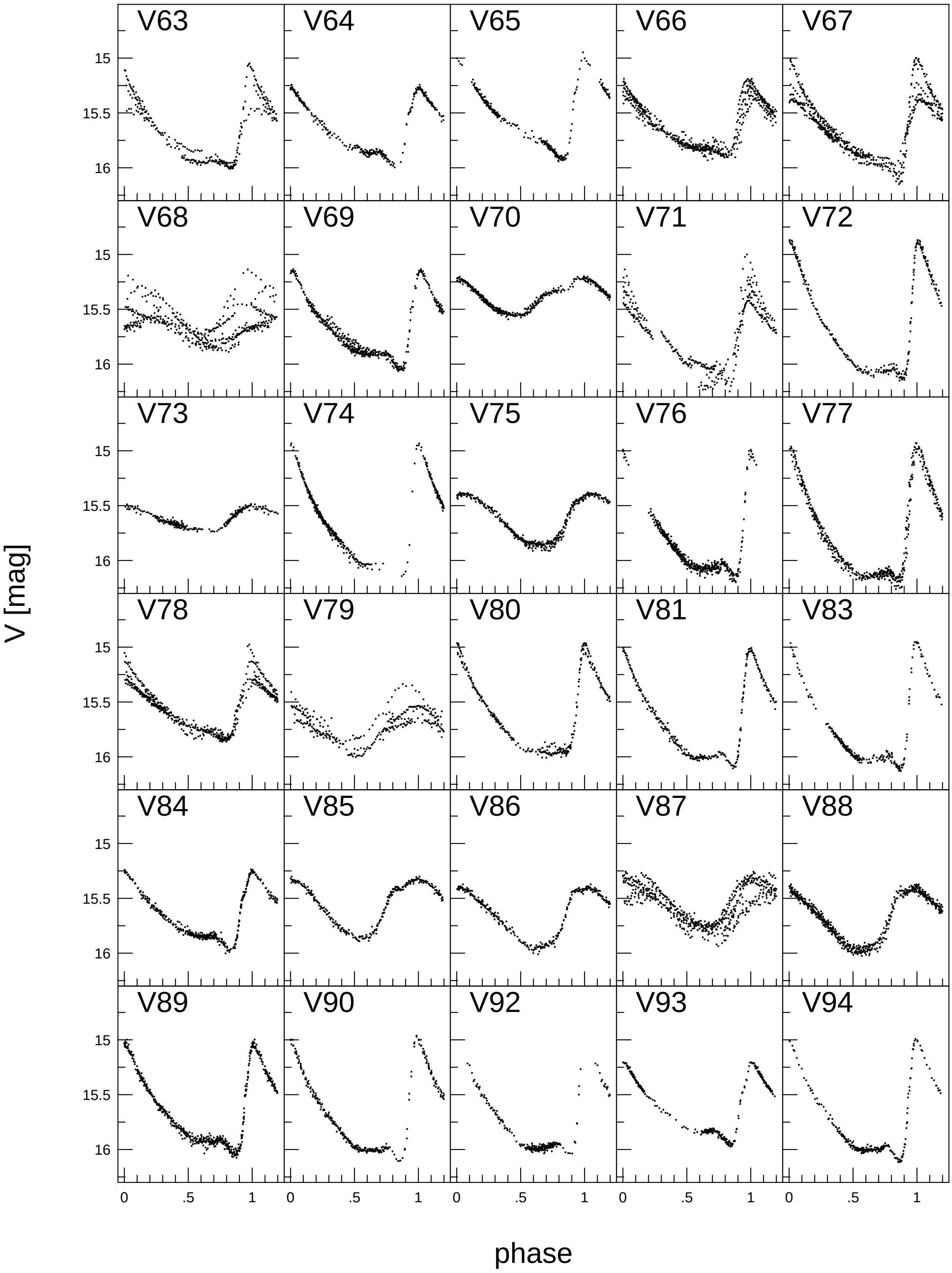


Figure 16. (continued)

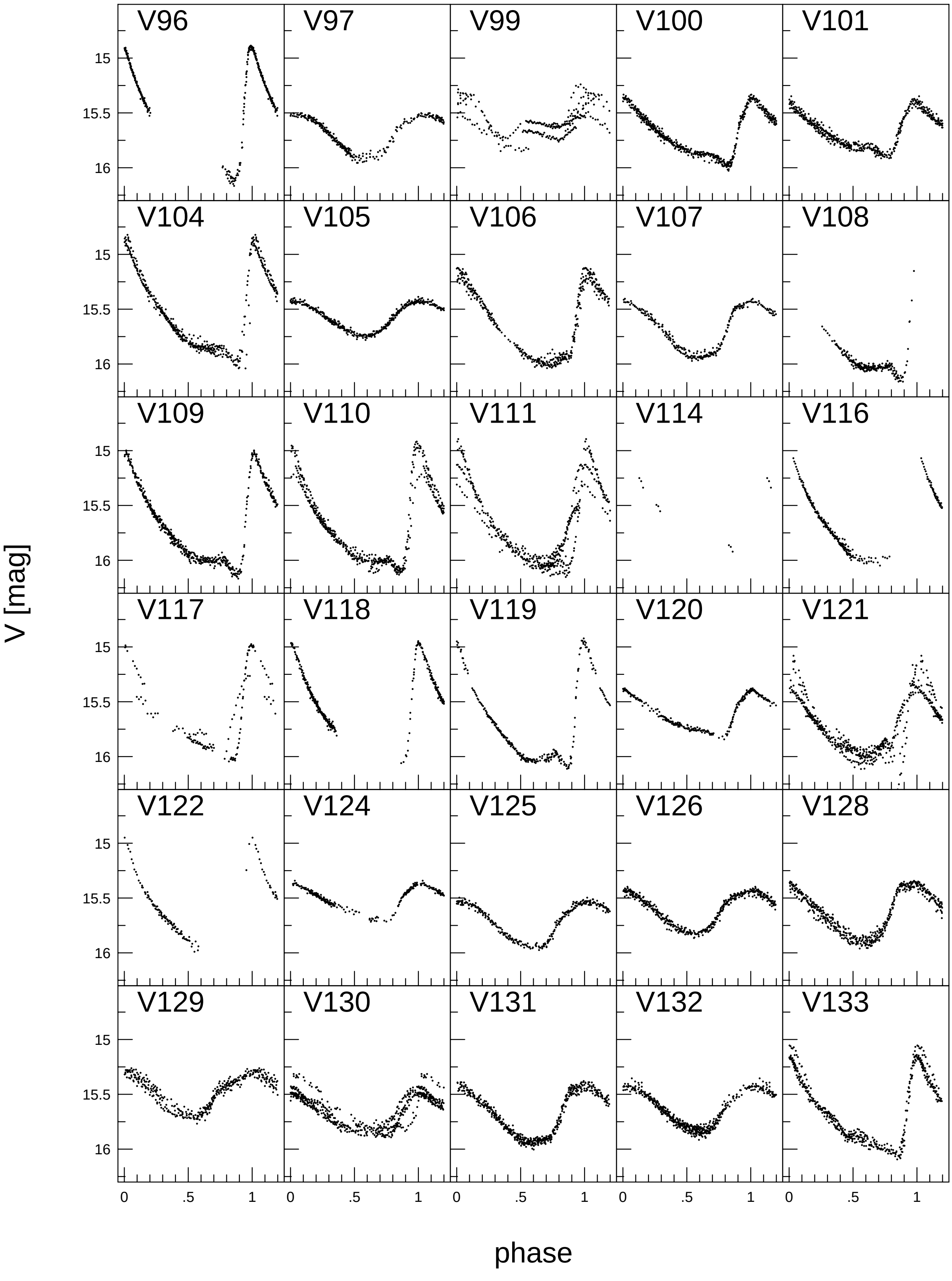
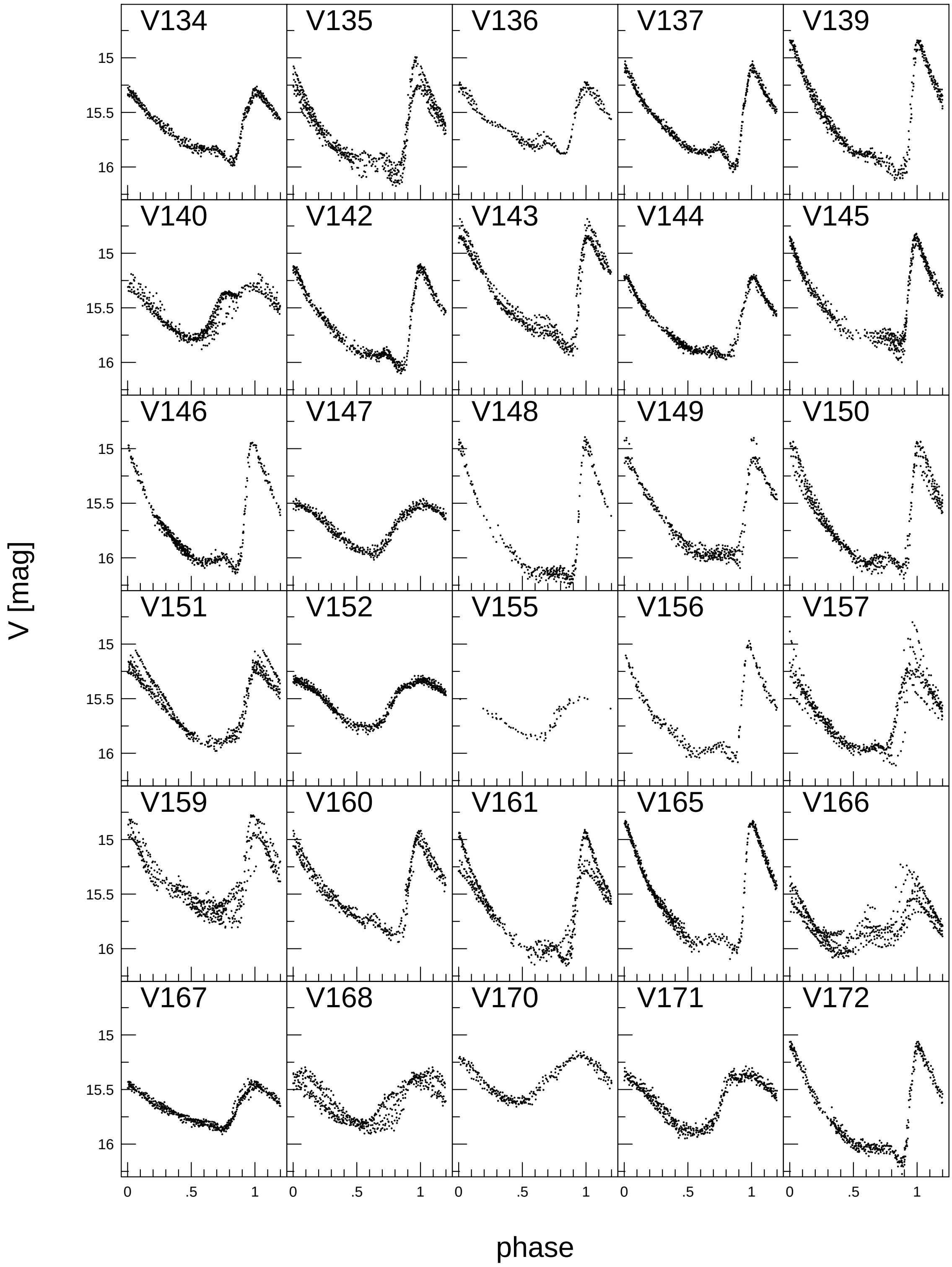


Figure 16. (continued)



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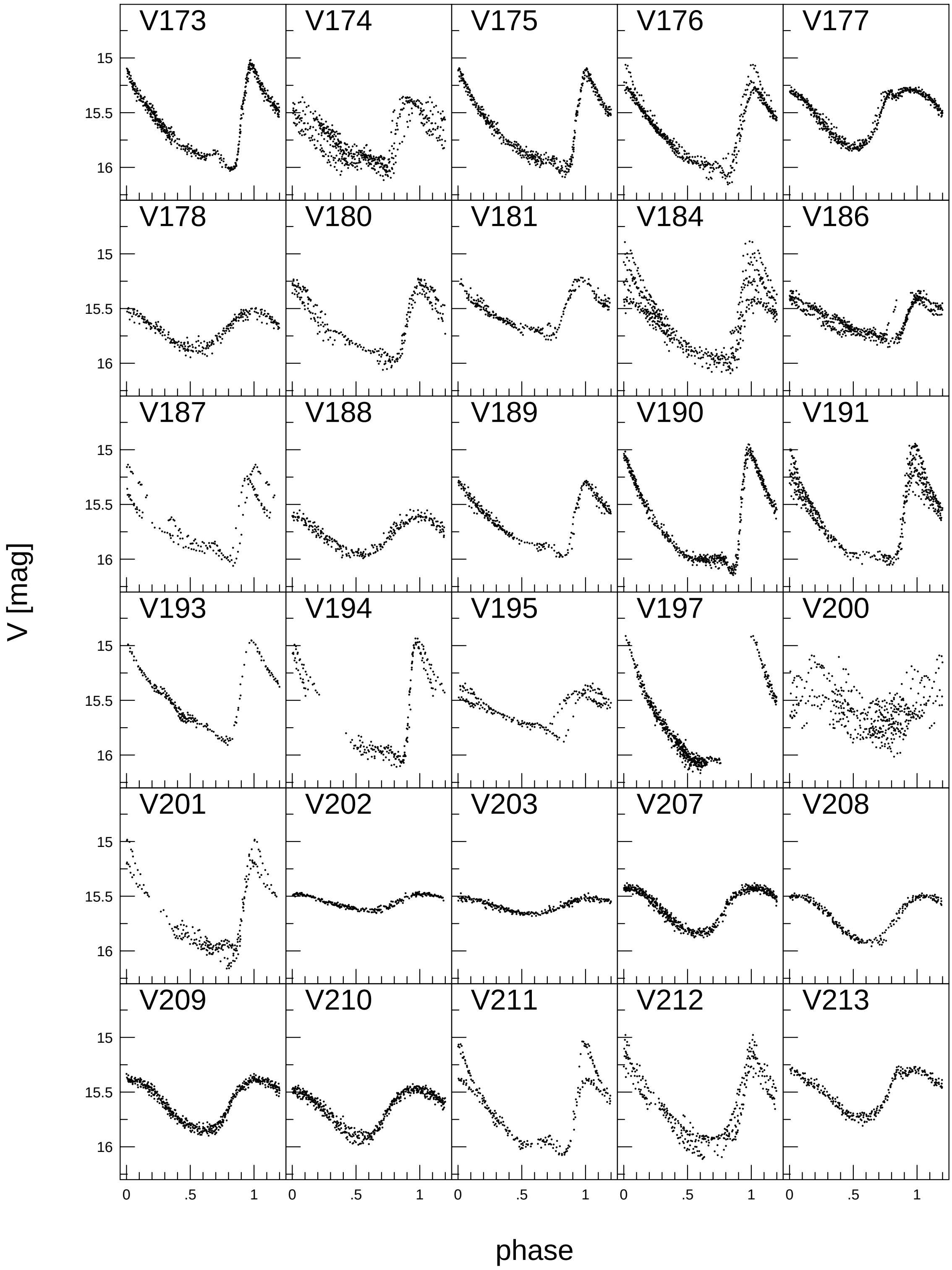

Figure 16. (continued)

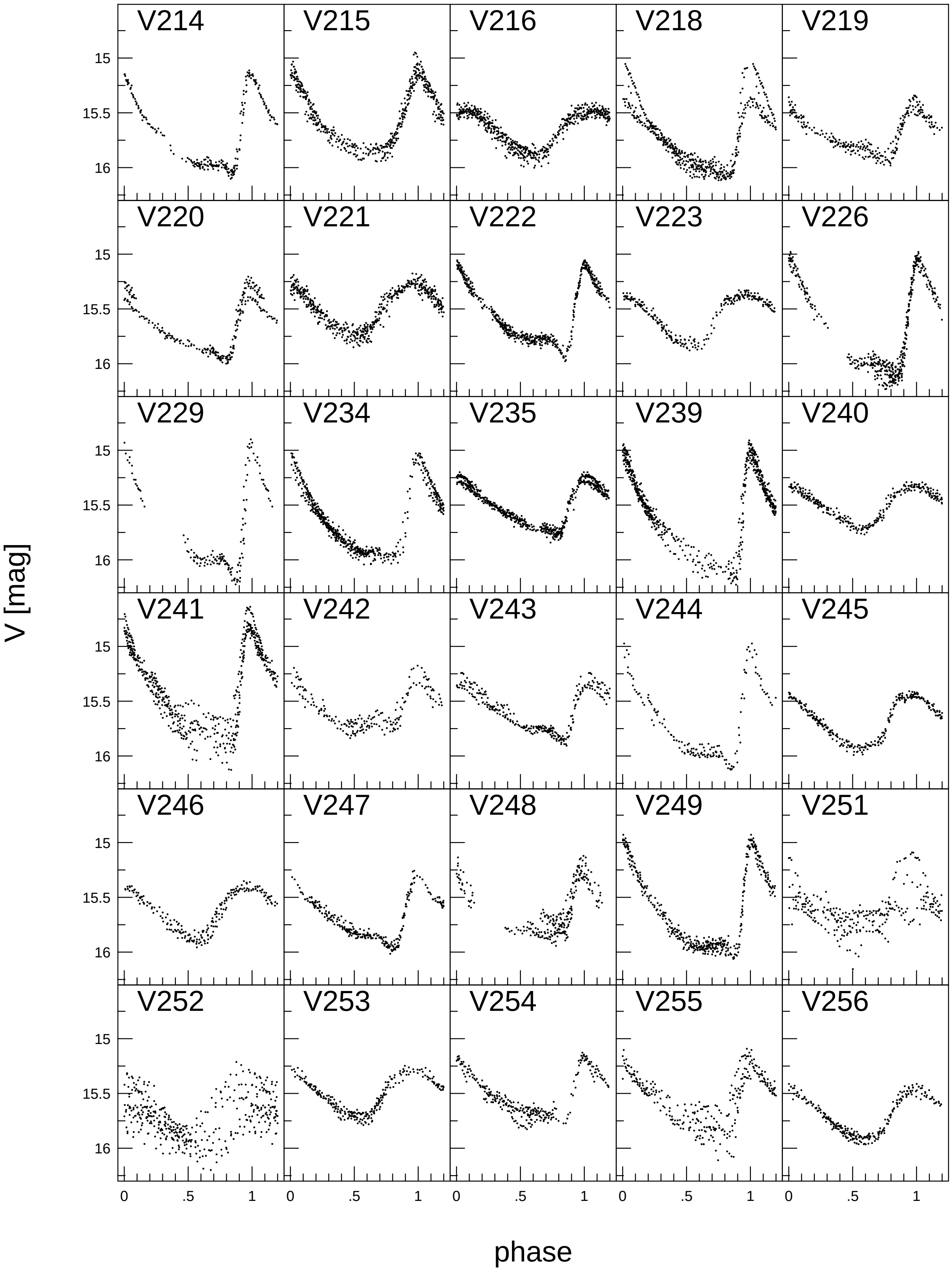

Figure 16. (continued)

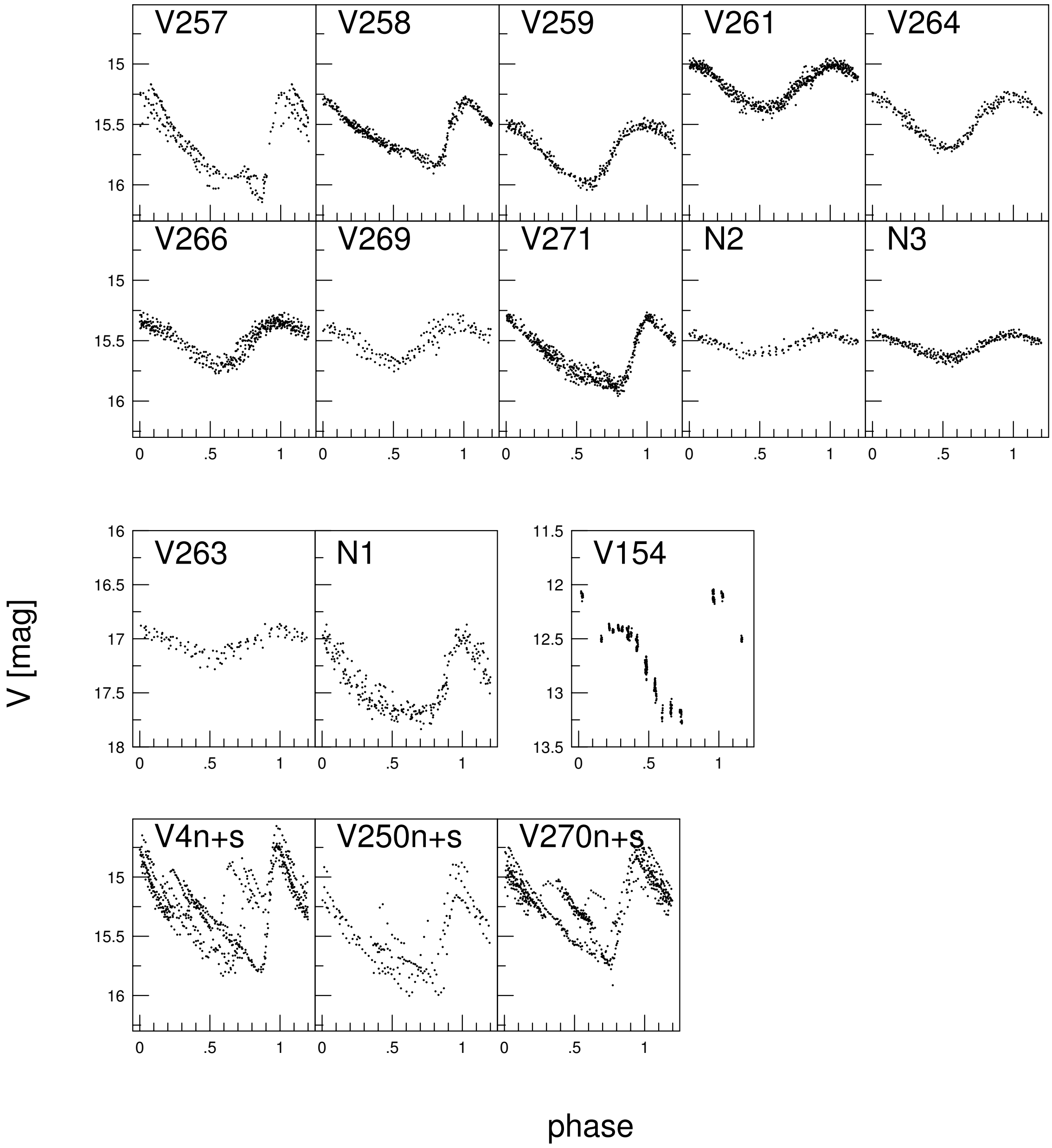

Figure 16. (continued)