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MITTEILUNGEN
DER
STERNWARTE
DER UNGARISCHEN AKADEMIE
DER WISSENSCHAFTEN

BUDAPEST-SZABADSÁGHEGY

Nr. 54

G. PAÁL

SOME REMARKS ON EMPIRICAL TESTS OF COSMOLOGY

BUDAPEST, 1964

ABSTRACT

In Abell's catalogue of clusters of galaxies [1] a strong correlation has been found between the compactness and the distance of the clusters (cf. Table 7). By comparing Abell's and Zwicky's [4] catalogues it is demonstrated here, that this results from an error in Abell's working hypothesis, according to which the ,,angular diameter'' of a cluster is inversely proportional to its mean red shift. Our result indicates that in the case of distant clusters of galaxies the red shift is not even in rough proportion to distance. This statement is in disagreement with the opinion generally held by astronomers, it is, however, proved to be in perfect accord with the magnitudes and red shifts published by Humason, Mayall and Sandage [9], provided the relationships between cosmologically important quantities are determined independently from Friedmann's equation generally used in cosmology. When the correct relationship between the red shifts and the angular diameters of the clusters of galaxies is applied, the existence of a metagalactic density gradient different from zero may be inferred from the data of Abell's Catalogue. The cosmological consequences of the points brought up in the present paper are mentioned in brief only, since they will be discussed in detail in a paper to follow.

SOME REMARKS ON EMPIRICAL TESTS OF COSMOLOGY

The data contained in Abell's catalogue of rich clusters of galaxies have been statistically evaluated in many respects by Abell himself [1]. The first attempt to analyse the clusters of galaxies according to their richness was made by K. Just [2]. The present study tries to develop the examination of the distribution of clusters of galaxies according to their richness and distance with the aid of some more recent data. Some new empirical relationships of cosmological significance are stated, and their possible consequences pointed out. As a first step Abell's work [1] is discussed in some details.

Abell's Catalogue of Clusters of Galaxies

Abell has considered „rich" and listed in his Catalogue all clusters of galaxies that could satisfactorily be distinguished on photographic plates taken with the 48-inch Schmidt telescope of the Mount Palomar Observatory meeting the following two criteria.

a) Δm or „richness" criterion: a cluster must contain at least fifty members that are not more than 2 mag. fainter than the third brightest member;

b) contour or „compactness" criterion: a cluster must be sufficiently compact that its fifty or more members are projected on the plate within an adequately defined contour of circular shape centered on the cluster image. The angular diameter, ϑ, of the corresponding „counting circle" in the sky has been defined by the expression $\vartheta = \frac{K}{\delta}$, where δ means the average red shift* and K is an empirical constant chosen suitably in the course of the measurements. Clusters defined in this manner have been grouped into distance and richness categories according to the photored magnitude of their tenth brightest member [1a, 3a] and according to the number of their members found within the angular diameter ϑ and the magnitude interval mentioned above, in the following way:

Table 1

Distance groups

Distance Group	1	2	3	4	5	6
Magnitude Range	13.3 – 14.0	14.1 – 14.8	14.9 – 15.6	15.7 – 16.4	16.5 – 17.2	17.3 – 18.0

* $\delta = \frac{\Delta\lambda}{\lambda}$, where λ is the laboratory wavelength of some spectrum line in the light of the galaxy, and $\Delta\lambda$ is its displacement owing to the red shift.

Table 2

Richness groups

Richness Group	I	II	III	IV	V
Counts	50—79	80—129	130—199	200—299	300 or over

Instead of the brightness of some representative members, the mean red shift, δ, of a cluster may also be used as a basis of grouping the clusters into distance classifications. The magnitudes listed in Table 1 are roughly equivalent to the following red shift values:

Table 3

Relation between Abell magnitudes and red shift

Magnitude (m)	14.0	14.8	15.6	16.4	17.2	18.0
Red Shift (δ).............	.030	.048	.076	.108	.148	.200

In his paper Abell has not examined the distribution of the clusters of galaxies according to richness for every distance group separately. The results of such analysis are summarised in Table 4a.

Table 4a

The distribution of the clusters of galaxies according to distance and richness (Entries of Abell's statistical investigations)

Richness Group \ Distance Group	1	2	3	4	5	6
I	5 55.6%	2 100%	26 78.8%	48 81.4%	518 78.9%	624 67.6%
II	4 44.4%	—	7 21.2%	9 15.2%	121 18.4%	243 26.3%
III	—	—	—	2 3.4%	18 2.7%	49 5.3%
IV	—	—	—	—	—	6 0.7%
V	—	—	—	—	—	1 0.1%

The Table indicates not only the number but also the percentage distribution of the clusters of galaxies among the various richness groups for each distance group separately.

Table 4a reveals a strong correlation between the distance and the richness of clusters of galaxies listed in Abell's Catalogue: in distance group 6 the abundance ratio of rich clusters of galaxies considerably increases. It is evident that in case of a constant abundance ratio, in each richness group the ratio of clusters belonging to distance groups 5 and 6 ought to be constant. The discrepancy in the value of these ratios related to the various richness groups can be used as a measure of the variation of the abundance ratio of the clusters. In Table 4b the strong variation of the abundance ratios found between distance groups 5 and 6 is characterized by these ratios.

Table 4b

Same as the two last columns of Table 4a, but the ratios of numbers of clusters in neighbouring distance groups are also indicated

Richness Group \ Distance Group	5		6
I	518	1.2	624
II	121	2.0	243
III	18	2.7	49
IV	—		6

K. Just was the first to call attention to this fact [2]. In his opinion the correlation between richness and distance is a proof for the evolution of the whole Universe, as, owing to the finite velocity of light, every information arriving from greater distances relates to former times. In the following part of the present paper this phenomenon is subjected to a critical analysis and the conclusions that may be drawn from it are pointed out.

The anomaly in the abundance ratio of clusters of different richness has been investigated and compared with a similar phenomenon that may be observed by studying the first volume of F. Zwicky's ,,Catalogue of Galaxies and of Clusters of Galaxies" [4]. (Further volumes have not been published so far.) The results of the investigation and their likely interpretation are discussed in the following Sections, each of them containing one statement and its relevant argumentation.

The results of investigation

1) *A definite correlation, of identical character with that resulting from Table 4a, is found between the richness and the distance of clusters of galaxies in various areas of the celestial sphere. This correlation, though not of equal strength, is striking in all sufficiently large regions of the sky so far investigated.*

This statement has been proved by the comparison of tables constructed in a similar manner as Table 4a, for various regions of the sky. For brevity's sake only a single Table, significant for further conclusions, is given here. Let the selected field of the sky be identical with that covered by the first volume of Zwicky's Catalogue: $-5° < D < 15°$; $7^h < RA < 18^h$ (Table 5).

Naturally, in the relatively poorer statistical sample of Table 5 there is a greater scatter. To reduce this, the number of rich clusters and that of clusters of smaller distances are represented together. A correlation between richness and distance different in amount, but of identical sense, can be clearly observed in the right-hand side upper part of the Table, which is statistically the most reliable one.

Table 5

The distribution of Abell's clusters according to their distance and richness in the area $-5° < D < 15°$; $7^h < RA < 18^h$

Richness Group \ Distance Group	1, 2, 3, 4 together	5	6
I	10 76.9%	68 88.3%	57 67.8%
II, III together	3 23.1%	9 11.7%	27 32.2%
IV, V together	—	—	—

2) *The observed number of clusters of richness group I divided by the number of clusters of richness group II in the first four distance groups goes increasing in a statistically less significant manner, whereas in the sixth distance group it decreases considerably.*

Before checking the above statement numerically, let us introduce the function N(m) commonly used in stellar statistics. Let the function N(m) regarding the clusters of galaxies represent the number of clusters found in the field under examination, in which the magnitude of the tenth brightest member is lower than the fixed magnitude, m. Considering that differences in the brightness of the tenth brightest member of clusters arise probably only from differences in their distances [1a], the function N(m) practically gives the number of clusters which are located nearer than a given limiting distance. It is also evident that in the knowledge of the function $m(\delta)$ (Table 3), the limit of count may also be expressed by distance parameter δ, and in this way the function $N(\delta)$ can be defined. Function $N_A(m)$ related to the entire Abell Catalogue, and $N_I(m)$, $N_{II}(m)$ ... showing the numbers of clusters belonging to the different richness groups, will also be introduced.

To check the statement of Section 2 it is sufficient to compare the ratios $\frac{N_I}{N_{II}}$ resulting from Table 4a (See Table 6).

The variation of the quotient $\frac{N_I}{N_{II}}$ can be best described by a function that increases as far as the fourth group, is fairly constant between groups 4 and 5, then abruptly decreases after distance group 5. As the first two or

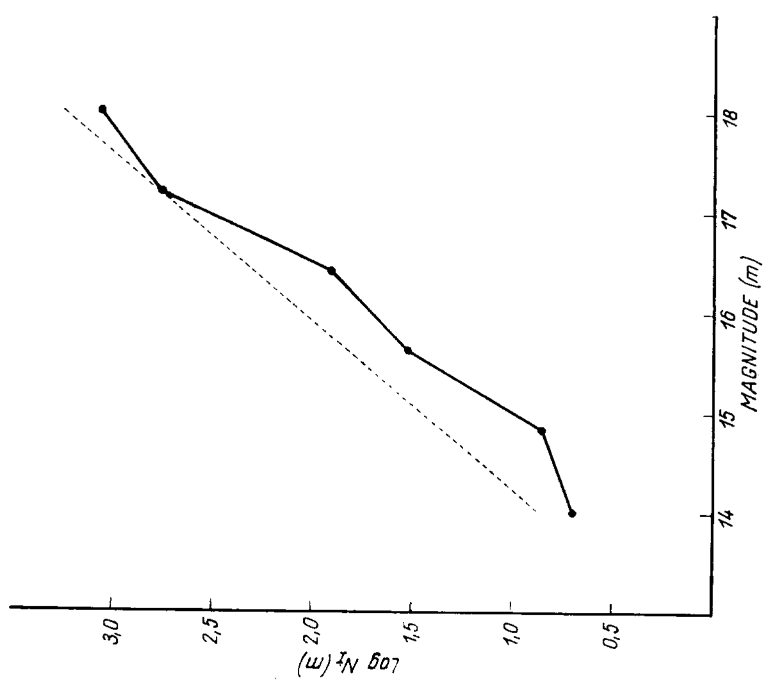

Fig. 1a. Function Log $N_{I}(m)$

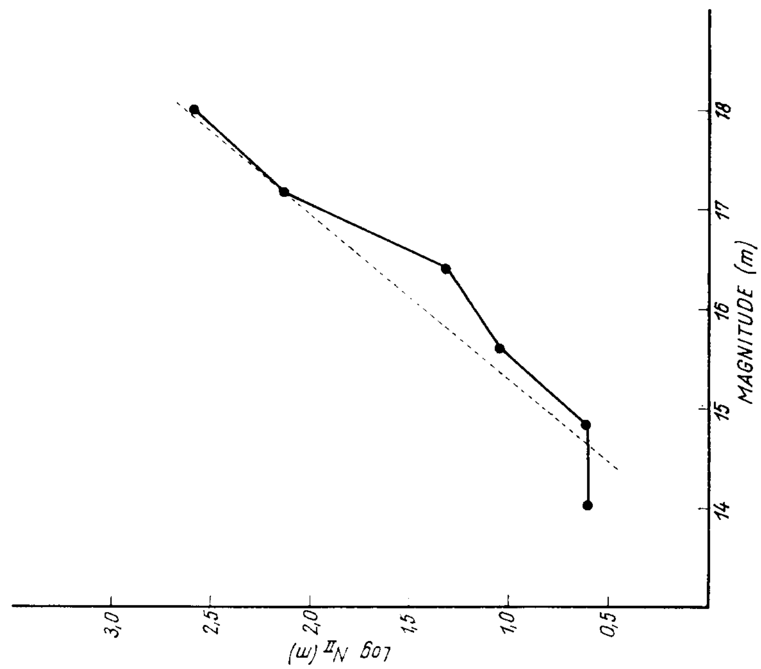

Fig. 1b. Function Log $N_{II}(m)$

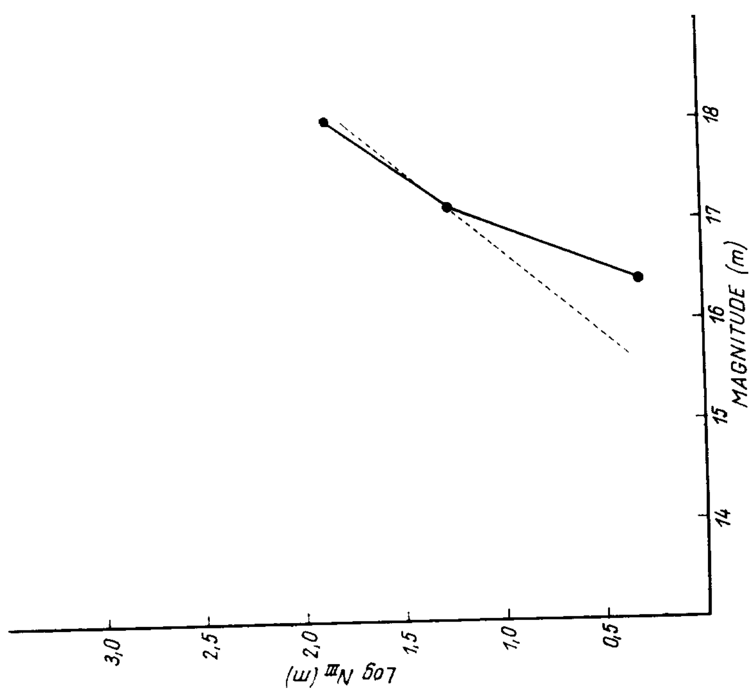

Fig. 1c. Function Log N_{III}(m)

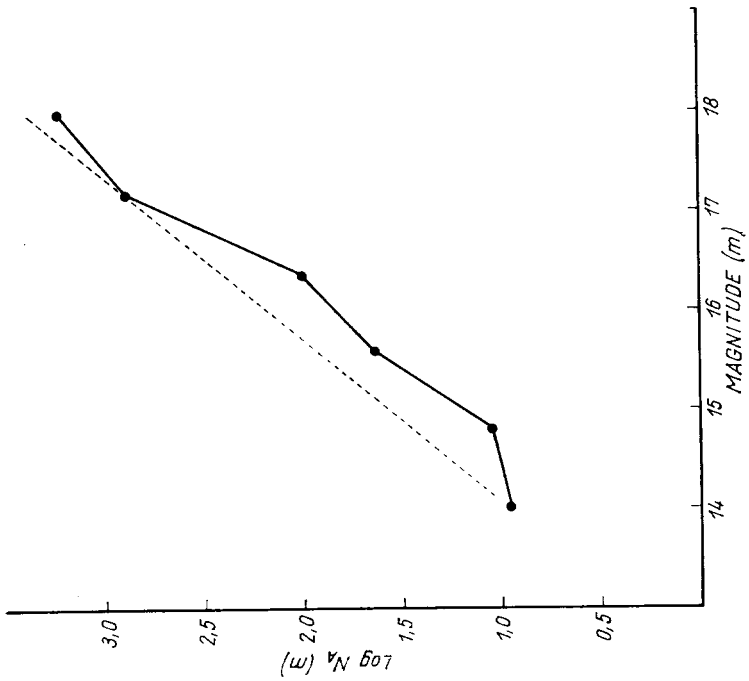

Fig. 1d. Function Log N_A(m)

three quotients are taken from poorer statistical samples, the possibility that the increase found in the first four groups is partially due to some random effect cannot be excluded.

Table 6

Ratio \ Limiting Magnitude	$14^m_\cdot0$	$14^m_\cdot8$	$15^m_\cdot6$	$16^m_\cdot4$	$17^m_\cdot2$	$18^m_\cdot0$
$\frac{N_I}{N_{II}}$	$\frac{5}{4} = 1.25$	$\frac{7}{4} = 1.75$	$\frac{33}{11} = 3.00$	$\frac{81}{20} = 4.05$	$\frac{599}{141} = 4.24$	$\frac{1223}{384} = 3.18$

The function Log N(m) referring to each richness group and the whole material separately are represented in Figures 1a, b, c and d.

These Figures clearly show the dependence on richness of the N-function. The dashed lines, expressed by the equation $N = 0.6\ m + \text{const.}$ in case of not too great distances correspond to the homogeneous space distribution of the clusters of galaxies [5]. The additive constants have always been selected so that the dashed line should fit the point corresponding to distance group 5. The curve Log $N_A(m)$ in distance groups 2, 3, 4 is nearly parallel to the dashed line of the slope 0.6. This suggests that the space distribution of the clusters in the volume of space under consideration and the statistical sample of the catalogue are fairly homogeneous.

3) *The correlation between the richness and the distance of the clusters results from a stronger correlation between the morphological type and the distance of the clusters.*

Taking Zwicky's morphological criteria [4, 6, 7] as a basis let us classify the clusters belonging to the various richness groups as compact, medium compact, and open clusters. It will be obvious that the correlation found on the right-hand side of Table 5 between the richness and the distance of the clusters is primarily due to a marked decrease in the abundance ratio of clusters of lower degree of compactness. To illustrate this statement, Table 7 gives the

Table 7

Abundance ratios of compact and medium compact clusters in richness group I

Type \ Distance Group	5	6
Medium compact Richness group I	44 68.7%	20 42.6%
Compact Richness group I	20 31.3%	27 57.4%

distribution of compact and medium compact clusters belonging to richness group I, between distance groups 5 and 6. In lack of other data, the comparison is restricted to the region of the sky included in the first volume of Zwicky's Catalogue.

4) *The correlation between distance and richness of clusters of galaxies observed in Abell's Catalogue does not originate in some phenomenon of nature, it results from the peculiarities of Abell's measurements used as a basis to the compilation of the Catalogue.*

This statement rests upon a comparative analysis of Abell's and Zwicky's Catalogues of clusters of galaxies.

The Zwicky Catalogue considers in its classification the morphological characteristics (degree of compactness) and the distance of the clusters of galaxies. It should be noted here that Zwicky's definition for distance groups differs from that of Abell discussed above. An arrangement in groups similar to that of Table 4a, worked out for this catalogue is given in Table 8.

Table 8

The distribution of clusters of galaxies listed in Zwicky's Catalogue, according to their compactness and distance

Degree of Compactness \ Distance Group	Near	Medium Distant	Distant	Very Distant
Compact	1 1.1%	14 6.8%	33 10.7%	130 30.6%
Med. Comp.	39 42.9%	99 48.1%	146 46.9%	202 47.5%
Open	51 56.0%	93 45.1%	132 42.4%	93 21.9%

In accordance with the composition of other tables, Table 8 indicates not only the number but, also the percentage distribution of the clusters of galaxies among the various morphological groups for each distance group separately.

From Table 8 it follows that the abundance ratio of clusters of lower degrees of compactness in Zwicky's Catalogue decreases with the distance. Confronted with those discussed in Section 3 this seems to be similar to the anomaly of abundance ratios found in Abell's data. The analogon of the statement of Section 1 applies also to the data contained in Zwicky's Catalogue, that is to say, a strong correlation between the type and the distance of the clusters appears also here, and is generally independent from the direction. There is, however, a definite divergence between the two Catalogues as regards the statement of Section 2. From the first distance group on the abundance ratio of compact to medium compact clusters in Table 8 increases with distance uniformly, strongly and strictly monotonously, contrary to the change definitely not monotonous and quite sudden found in Abell's Catalogue. This can be better followed on Log N diagrams plotted for Zwicky's types of clusters, represented in Figures 2 a, b and c.

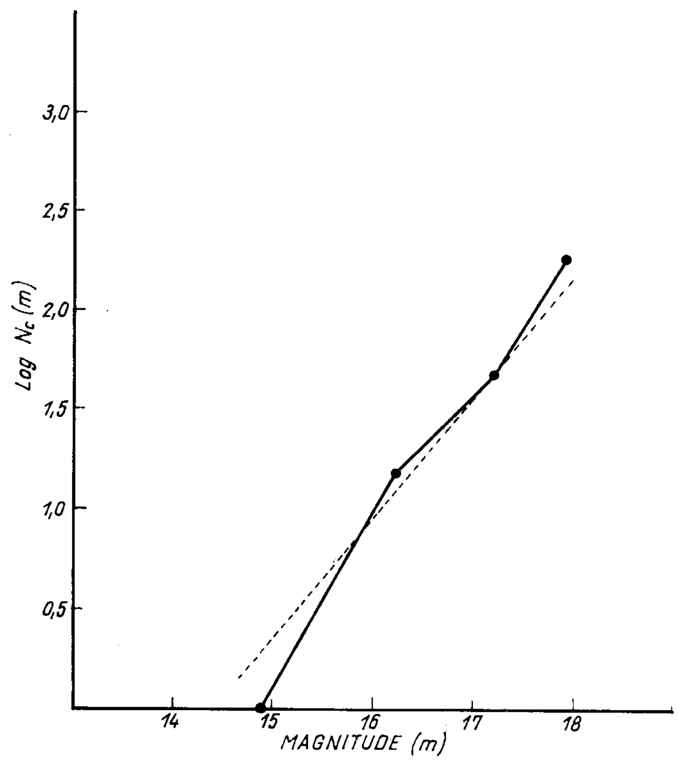

Fig. 2a. Function Log $N_c(m)$ plotted for Zwicky's compact clusters

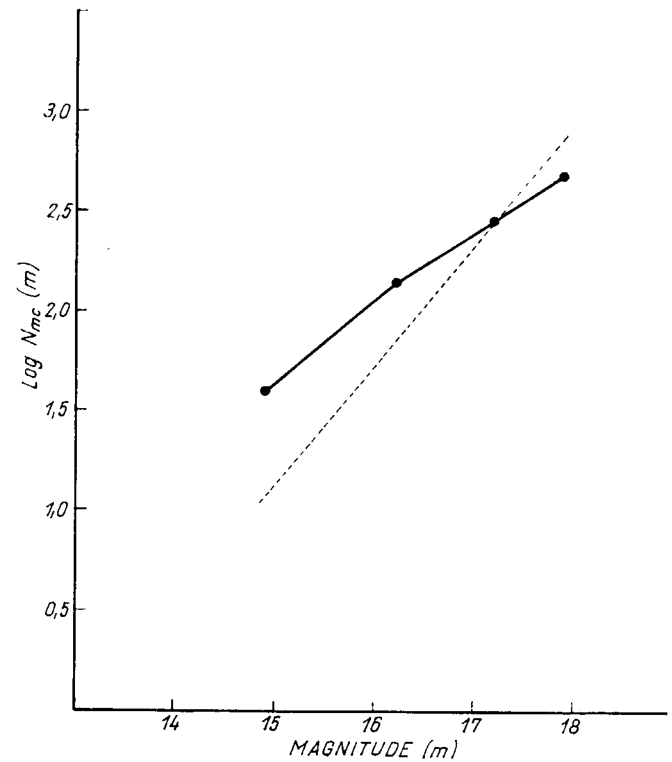

Fig. 2b. Function Log $N_{mc}(m)$ plotted for Zwicky's medium compact clusters

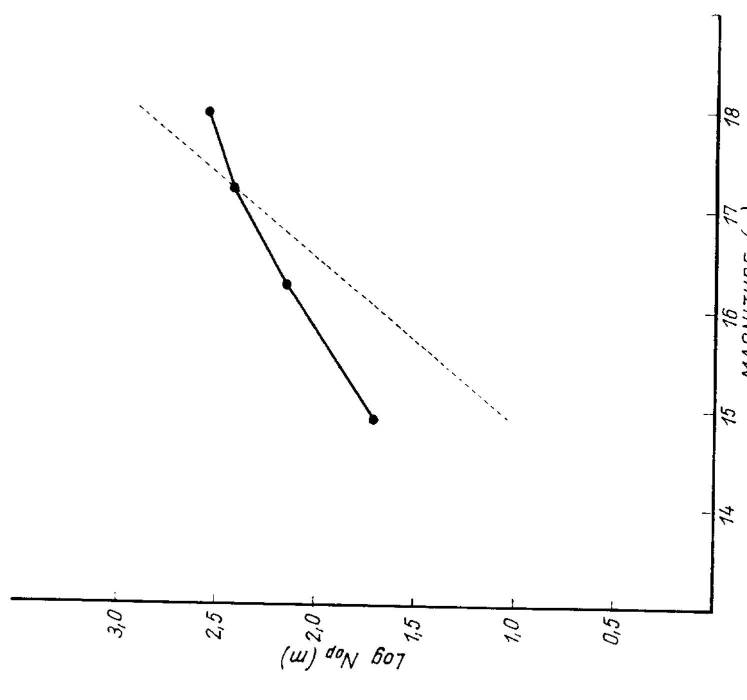

Fig. 2*c*. Function Log N_{op}(m) plotted for Zwicky's open clusters

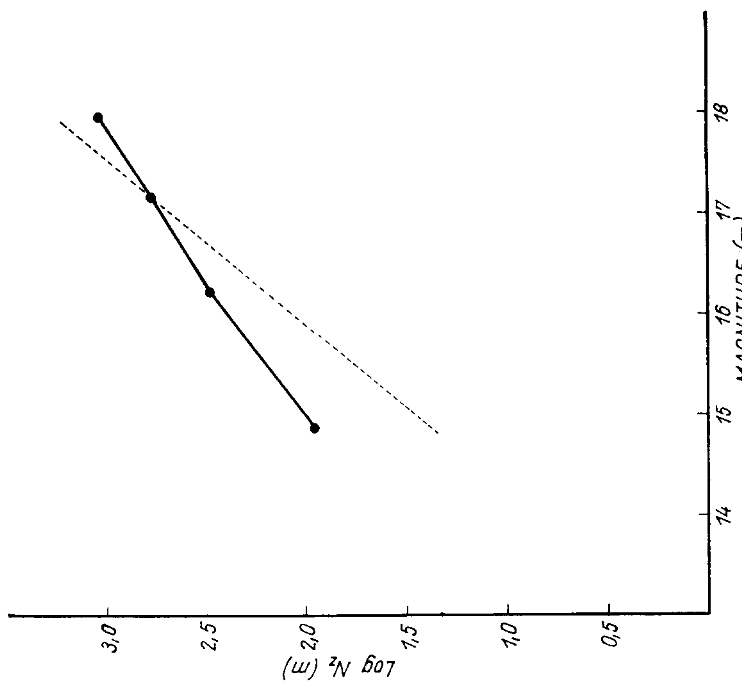

Fig. 2*d*. Function Log N_z(m) plotted for all clusters of galaxies included in Zwicky's Catalogue

Zwicky has defined the distance groups by red shift values. They are expressed here by Abell's photored magnitudes.

From Figures 2 a, b and c it is clear that the apparent variation in the abundance ratio of clusters of various types does not come from compact clusters becoming more frequent with the distance, but it originates in apparent decrease of the space frequencies of medium compact and open clusters. Considering that medium compact and open clusters are more frequent in space, the $N_Z(m)$ diagram plotted for the entire material of Zwicky's Catalogue will obviously strongly deviate from that which could be expected in case of a homogeneous space distribution beginning with the very first distance group (Fig. 2d).

It follows from this that if Zwicky's Catalogue is approximately complete, the distribution in space of the clusters of galaxies is inhomogeneous; beginning from the nearest distances, the actual density of the space distribution of the clusters significantly decreases. It is evident, however, that a real natural phenomenon of this kind should manifest itself also from the very first point of the $N_A(m)$ diagram, which is not the case in Figure 1d. *Consequently, in Zwicky's Catalogue there must be some kind of selection in the measurements.* Starting from other considerations, Abell has examined this selection in detail. According to his results, the ,,contour'' within which Zwicky counts the members of the clusters, systematically varies with the distance, and this variaton largely depends on the type of the cluster of galaxies under consideration [8a]. The effect of the selection is that at greater distances generally less galaxies are counted in the clusters, and owing to this, the number of objects considered as clusters goes decreasing. In case of compact clusters the decrease is not too considerable, but in case of more diffuse clusters it becomes very significant. This is the source of the variation of the abundance ratio in Zwicky's Catalogue. Particularly in the case of clusters of lower degree of compactness with the increase of distance an increasing selection, that is to say, a LogN(m) curve of decreasing slope could be expected. This is shown in Figures 2b and 2c. Most probably, the apparently monotonous variation of the abundance ratio in Zwicky's Catalogue must be ascribed to a single effect, which means that the results of the Catalogue are incompatible with the assumption that in nature the real density of the space distribution of medium compact clusters at great distances undergoes an abrupt change. Contrary to the facts observed, such a change, superposed to the independent selection-effect, should be easy to observe. Abell's Catalogue shows an abrupt change of about three times in the abundance ratio of compact to medium compact clusters between distance groups 5 and 6 in the field of the sky examined by Zwicky (cf. Table 7). *Considering what has been said above such a variation cannot exist in reality, it must come from some systematic error in Abell's measuring procedure.* A numerical comparison shows that the anomaly in Abell's Catalogue, if it were a real one, could offer a satisfactory explanation in itself for the surprisingly mild slope of the final section of the Log $N_Z(m)$ curve. But this supposition excludes the existence of the Zwicky selection which, however, does exist as it manifests itself more and more distinctly with the increasing distance (cf. the slopes of curves 1a, b, d and Table 7). It should be noted that Zwicky's Δm and contour criteria both differ from that of Abell's owing to which Zwicky's Catalogue includes also clusters of lower population. It is *per se* unlikely that the anomaly in the abundance ratio of smaller clusters is compensated by clusters of even smaller population. Yet, for a check, clusters of adequate richness have been selected from Zwicky's Catalogue and their distribution according to types compared to the distribution of those included in Abell's Catalogue. Also this investigation has led to the above conclusion.

In conclusion it may be stated that the data published in both catalogues of clusters of galaxies are affected by significant systematical errors. For Zwicky's Catalogue the source of the error has been detected. In the following an attempt is made to trace the origin of the selection manifesting itself in Abell's Catalogue.

5) *An analysis of Abell's measuring procedure leads to the conclusion that the correlation between the richness and the distance of the clusters results from the inadequacy of Abell's convention for the angular diameters of the counting circle, which reads* $\vartheta = \frac{K}{\delta}$.

Let us consider the possible sources of errors one by one. Errors may come from 1) an inadequate *identification* and from 2) an erroneous *classification* of clusters into richness and distance groups. Both sources of errors may be either systematical or random. As the origin of a systematical, tendency-like effect is looked for, random errors are of no importance for the present investigation. Abell himself has investigated possible errors in his identification-procedure [1b]. According to his results, these cannot amount to more than 1—2 per cent. Systematic errors in classification, on the other hand, should be investigated separately to see whether they can produce relative effects depending on the type of the clusters under examination.

Undoubtedly, the factors influencing the measurements as instrumental errors, intrinsic and external inhomogeneities of the photoplates, atmospheric extinction and interstellar or intergalactic absorption cannot result in favouring a certain type of clusters to the detriment of the others. It is more difficult to assess the effects of using in the definition of clusters unequal magnitude intervals or inadequate angular diameters of the circles within which cluster galaxies are counted.

Let us first survey the systematical errors in the scale of magnitude.

There is a considerable chance that in morphologically different clusters member galaxies of different types are photographed. First of all, it should be elucidated whether the error in the observed magnitude can depend on the type of the galaxies in question.

Against the supposition, that the errors in magnitude would depend on the type of galaxies under examination advocates the fact that there is a one-to-one relation between the brightness and the mean red shift of clusters of galaxies, without regard to the type or richness of the cluster. This is shown by Abell's $m(\delta)$ relationship [1c], represented in Figure 3 with the indication of the degree of richness of the clusters of galaxies used for calibration.

The natural scatter showing in Figure 3 can hide a difference of about one tenth of a magnitude between the $m(\delta)$ relations of clusters of different types, but a relative effect in counts can come only from a differential error in the magnitude scale, which, evidently, must be smaller by one more order of magnitude. (It should be remembered here that Abell's distance groups

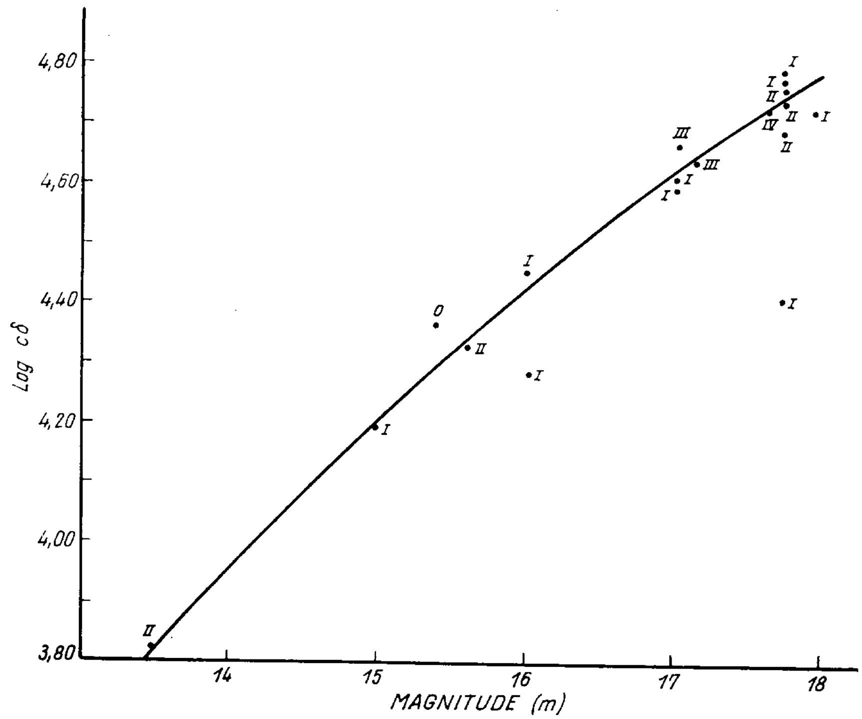

Fig. 3. Relation between Abell's photored magnitude and red shift. Figures next the points indicate the richness group of the clusters of galaxies. The ordinate bears the logarithm of the product of the red shift and the velocity of light (After Abell [1])

are defined with an accuracy of $0^m.1$, and the probable error in individual magnitude determinations is about $0^m.2$.) For completeness' sake it should be mentioned that magnitudes measured at great distances can change independently from possible errors of measurements also because of the red shift appearing in their spectra. This effect is compensated by the K-correction. Theory holds that the value of the K-correction is slightly dependent on the type of galaxies and hence the differential magnitude correction is of the order of $0^m.01$ magnitude in the spectrum range in which Abell made his measurements [9a, 10a, 11a].

We have to assume therefore that the Abell scale of magnitude can be subjected only to distortions independent from the type of the galaxies examined. Such an error of magnitude may influence the limits of the distance groups, but not in a manner dependent on the richness or on the type of the clusters of galaxies. Through the error in the convention $\Delta m = 2$ (cf. p. 4) the error in magnitude affects the count of the members of the clusters, yet, as shown in Figure 4, a small variation of the measured interval of magnitude (of maximum $\pm 0^m.1$) brings about an identical and small relative change (of maximum 6—7 per cent.) in the counts of galaxies in the various clusters

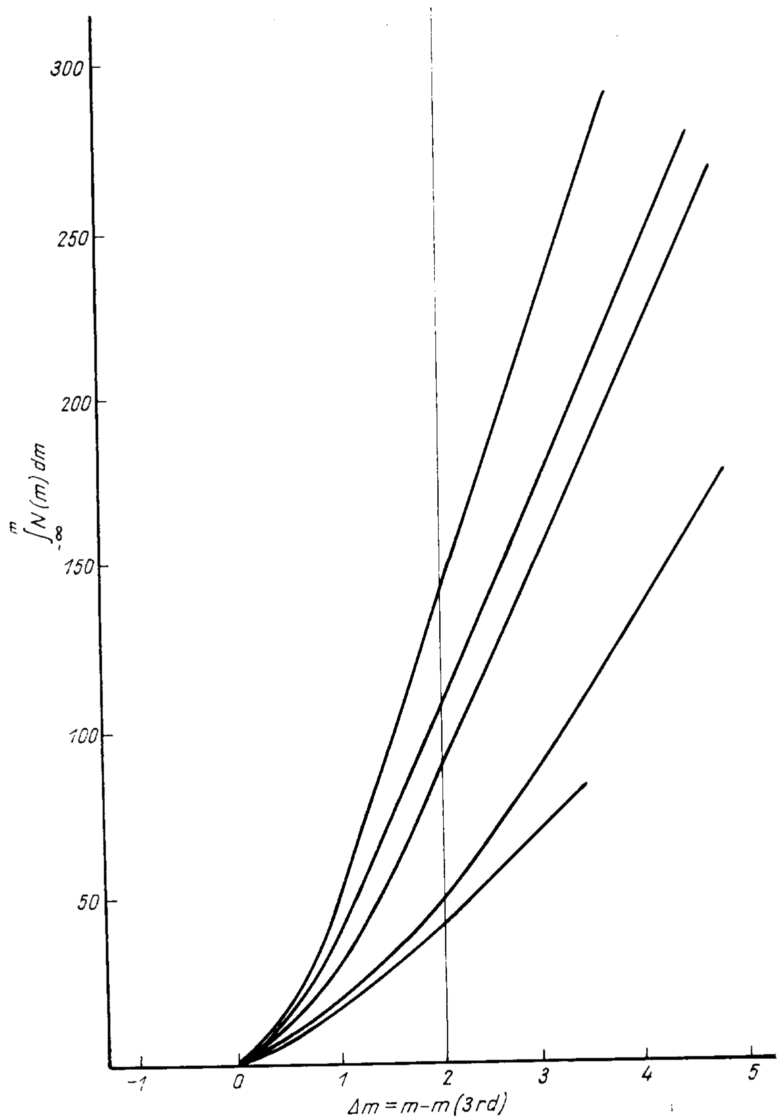

Fig. 4. The dependence of the counts of galaxies in clusters of different richness on the value Δm adopted in the counting. The five curves relate to five clusters of different richness. (After Abell [1], Fig. 2.)

thus producing only slight and fairly equivalent changes in the abundance ratio of the various types of clusters. Therefore, an error in the magnitude cannot have any significant relative influence.

This conclusion is supported by a further consideration. According to Abell's discussion the presence of an error of magnitude of technical origin in the data of the catalogue is most probable [1d], it is, however of some importance only for galaxies not very far away, and manifests itself in the reduction of Δm, whereas for galaxies in distance groups 5 and 6 only a slight effect contrary to the above may be expected (cf. the $m(\delta)$ curves of papers [1] and [9]). On the other hand, the anomaly in the abundance ratio becomes significant just between distance groups 5 and 6 and is not contrary in sense to the less significant change, found at the smallest distances.

It appears that the systematic variation in the abundance ratio of various types of clusters of galaxies discussed in Sections 1, 2, 3 and represented in Table 4a cannot originate in any error of measurement, except the convention regarding to the angular diameters of the counting circles. Considering what has been said in Section 4, it may be stated that Abell's convention must be in error.

Let us examine the effects that may be expected if for clusters at different distances the angular diameters of corresponding sizes are correctly described by function $\vartheta(\delta)$, greatly differing from $\vartheta = \dfrac{K}{\delta}$.

Abell wanted to limit the counts with angular diameters rather narrow, covering only the more dense central parts of the clusters. He has found a suggestive expression for the condition that in this inner space the number of galaxies, n should be fifty or more, calling it ,,compactness criterion". The clusters belonging to different Abell richness groups are classified on the basis of the same number n, that is to say, practically on the ground of the degree of their central density or their compactness. Though Zwicky defines the degree of compactness of clusters by characteristics to be observed in the innermost, still narrower central parts of the galaxies [4, 6, 7], it is natural that a statistical investigation finds a very close positive correlation between the two kinds of concept of ,,compactness". According to our examination Abell's richness group III, that includes highly compact clusters, consists of clusters considered compact also in Zwicky's sense without exception; Abell's group II consists of such only for the greater part; and group I predominantly includes clusters of other types, classified by Zwicky as medium compact or as open.

Let us consider first the case when with the growing distance the counting circle defined by Abell covers more and more restricted parts of the clusters. In this case Abell's counting procedure imposes a ,,compactness criterion" becoming more and more limited with the growing distance of the clusters observed. Less compact or more diffuse clusters meeting the criterion to an ever smaller extent will get gradually omitted from the catalogue. Table 6 reveals the presence of this omission in the catalogue.

By applying this procedure a number of clusters fail to enter the various richness groups. The higher the percentage of non-compact clusters, the higher the ratio of misclassified ones. Consequently, this effect should show best in the first richness group — in complete accordance with the data listed in Table 4a. It is quite obvious that the diminution of the number of clusters in richness group I cannot be counterbalanced by the misclassification of a corresponding number of clusters in richness group II, which is poorer in clusters and sustains a lower relative loss.

According to experience and to all cosmological theories in the first three distance groups the red shift may be considered proportional to distance [3b, 12a], and in this case Abell's convention regarding the counting circle must be right. An excessive restriction of the counting circle can be significant and of a rapid rate only in the farthest distance groups.

The opposite of this error in the angular diameter will not be examined here in detail. In the apparent abundance ratio of the clusters it would produce the opposite effect and would clash with the facts observed. It would also be completely irreal from the point of view of theory [10b].

Considering the connection between the apparent loss of members of clusters and the loss of members of richness groups on an illustrative numerical example let the observed numbers of galaxies in medium compact clusters belonging to distance group 6 and to richness group I be lower by, say, 20 per cent. than those belonging to distance group 5. (In reality, only the average loss of members can be expressed by a single number.) On the other hand, let us assume that the systematic observational error does not significantly influence the number of members counted in compact clusters in the same distance and richness groups. The clusters considered meet the criterion $50 \leq n < 80$ (where n is the number of galaxies counted). In accordance with our assumption each cluster classified as medium compact, whose number of members in the distance group 5 would have been $50 \leq n < 63$, would have failed to be counted owing to an increased observational error. The loss reaches 43 per cent. of the possible values n for richness group I. Yet, in accordance with a number of other independent observations, Table 4 shows that the space frequency of the clusters rapidly grows as n decreases. Owing to this fact the number of medium compact clusters in which $50 \leq n < 63$ is not 43 per cent. of the total amount of medium compact clusters in richness group I, but considerably higher.

As a result, we find that by failing to count only 20 per cent. of the galaxies in each cluster classified as medium compact, in the first richness group the majority of medium compact clusters gets lost. A similar operation taking 30 per cent. as a basis shows that more than 72 per cent. of the medium compact clusters in richness group I would be lost. This, would probably result in an anomaly in the apparent abundance ratio, exceeding the observed one. *Thus the apparent loss in members of the first richness group is a very sensitive indication of small systematic losses in the counts of the membership of the clusters.*

Abell himself has made informative investigations to reveal the extent to which a possible error in the angular diameter could influence the count of galaxies in clusters. (He does not mention any investigation referring to the number of clusters in richness groups.) In his opinion „the recorded „richness" of a cluster would seldom be affected significantly if the radius of the counting circle were changed by 10 or 15 per cent." [8b]. Though the probable relative error in the radius of Abell's counting circle is more than 15 per cent. (cf. remark d in the present Section), at the first glance this statement seems to be hardly reconciliable with what has been said above. Let us take a numerical example again. Supposing the erroneous reduction of the counting circle between distance groups 5 and 6 makes the counts in the clusters of more diffuse consistency drop by an average of about 20 per cent. It has been demonstrated above that such a systematic effect can produce radical changes in the numbers of clusters found in the various richness groups. Yet the change in the numbers of members in individual clusters being only at the limit of accuracy of the count, is practically untraceable, and Abell is right in terming it ,,insignificant". To prove this we quote Abell [1e]: ,,The counts of membership of a cluster, intended as richness criteria, are approximate only. It was desirable, therefore, to group the cluster into categories according to their richness in such a manner that a negligible number of clusters would be misclassified by *more than one* group interval. The standard error of an individual count was estimated at about 17 per cent." It is seen that Abell considers the misclassification of clusters into the next richness group still tolerable. Our investigation, on the other hand, is critically influenced just by the systematic occurrence of this phenomenon. *This means, that for the detection of otherwise untraceably small systematic errors in the counting of galaxies, and through this, for the determination of the error in the convention regarding the angular diameters, a new method of high sensitivity has been developed.*

The following points should be noted here:

a) The objects investigated by Abell for the determination of the change in count were, naturally, not without exception medium compact clusters belonging to distance groups 5 and 6, it even might be that not a single such cluster was among them. A narrowing down of the radius of the counting circle by 15 per cent. might significantly influence such clusters also individually.

b) Abell also admits in principle that the strong correlation between distance and richness might be the result of some small selection in the measuring procedure [8c].

c) Naturally, it does not follow from the above that besides the error in the determination of the counting circle no other error in observation or no natural phenomenon can influence the anomaly in the frequency distribution observed to a smaller extent.

d) It may be proved independently from the Einstein equations that provided the magnitudes of the synthetically brightest members of the clusters of galaxies [1a, 3a] do not show systematic deviations larger than one magnitude in the ,,world picture" [13], in conformity with the relativistic theory of the propagation of light Abell's convention regarding the angular diameters is certainly erroneous. Between the end points of distance groups 5 and 6 a relative variation of about 25 per cent. of the actual diameters (in space) defined by counting circle may be expected.

A short train of ideas proving the above statement is given in the Appendix.

6) *The function $\vartheta(\delta)$ giving the correct angular diameters for the corresponding parts of the clusters of galaxies can be determined to a reasonable accuracy for the interval $0.16 < \delta < 0.20$ by repeating a small fraction of Abell's measurements with the use of considerably modified counting circles.*

The angular diameter ϑ can be defined by any corresponding dimension of clusters of galaxies of any type. This means that the function $\vartheta(\delta)$ is undefined to a constant factor. From those discussed in Section 5 it is known that if Abell's measurements are repeated by using any of the correct functions $\vartheta(\delta)$, no strong apparent correlation can result between the type and the distance of the clusters. Based on this remark, a method permitting the approximate determination of the correct $\vartheta(\delta)$ function for small distance ranges may be developed.

In distance groups 5 and 6 δ does not change considerably. Through this short interval the function $\vartheta(\delta)$ may safely be approximated by a single linear function. According to experience, the field of clusters covered by Abell's counting circle from the fifth distance group on is reduced to such an extent that its further restriction would significantly affect the count, and hence the observed abundance ratio of the clusters of galaxies. For this reason the undetermined factor in function $\vartheta(\delta)$ should be selected so that in case of a value δ belonging to distance group 5, Abell's function $\vartheta = \dfrac{K}{\delta}$ should coincide with the real function $\vartheta(\delta)$ that is to say $\vartheta(\delta_5) = \dfrac{K}{\delta_5}$.

To approximate the function $\vartheta(\delta)$ thus defined, take a few adequate linear functions, ϑ_i, all fulfilling the condition $\vartheta_5 = \vartheta_i(\delta_5)$, and use them in turn to count the members in the clusters belonging to distance groups 5 and 6, leaving, of course, all the rest of Abell's conventions unchanged. Go on with this procedure until the apparent correlation between the richness and the distance of the clusters ceases to appear.

This programme, rather tiresome, may considerably be simplified without the results being significantly affected and can be executed on photoplates taken with smaller Schmidt telescopes if the task is reversed. Instead of experimenting with the enlargement of the counting circles to eliminate the anomaly manifesting itself in distance group 6, which is under a strong selection effect, counts should be made in distance group 5, more convenient for observation, by using adequately reduced counting circles to produce the anomaly observed in distance group 6. In this case the observations may be restricted to distance group 5. The extent of the required reduction may be determined in first approximation after the examination of a few scores of clusters. This process artificially produces the effect of selection in the measurements, and thus the error in the Abell formula, $\vartheta = \dfrac{K}{\delta}$ may be revealed.*

7) *The function $N_{corr}(m)$, corrected for error in the counting circle for the first five distance groups reasonably agrees with the one published by Abell, but in distance group 6 it becomes even steeper than function $N_{III}(m)$.*

The correctness of this statement is very easy to control. Owing to the excessive reduction of the counting circle, discussed in Section 5, the counts of galaxies belonging to a cluster decrease as the distance increases. In accordance with theory and observations, this effect goes increasing with the distance, which means that it makes the slope of function Log N(m) milder. Though this effect is significant only for more diffuse clusters, to a smaller extent

* A recounting of the clusters making part of Abell's distance groups 3, 4, 5 is planned on plates taken with the 60/90/180 cm Schmidt telescope of the Mountain station of the Konkoly Observatory and on copies of the National Society Palomer Sky Atlas.

it necessarily appears also in the case of highly compact and rich clusters. This means that the curve corrected for the selection must be even steeper than the one representing the function Log $N_{III}(m)$. The latter function is known to the required statistical accuracy only between distance groups 5 and 6. In smaller distances, however, the extent of the restriction of the counting circle and its effect on the outer, less compact parts of the clusters are of no great significance (cf. Section 2), and thus it may be substituted by the Log $N_A(m)$ curve with a fair approximation. The corrected curve is shown in Figure 5.

The correctness of the Log N(m) curve, made steeper in its final section, is also indicated by the evenness of its course. In opposition to those of the original curve, the points of the corrected diagram display only a very slight scatter and are located along a mild curve of a uniform bend (cf. also Section 9). It is worth to mention that the statistical weight of the individual points in Figure 5 rapidly increases from the left to the right, and therefore the points of the curve on the left-hand side of the Figure may show some scatter, whereas those on the right-hand side only systematic deviations. In all probability, we have succeded in correcting the latter in the first approximation.

8) *Further points advocating the function Log $N_{corr}(m)$ of greater steepness may be found in the parts of Abell's Catalogue statistically not investigated by the Author himself and also in Zwicky's Catalogue.*

Abell's Catalogue covers the entire area of the celestial sphere that may be observed from the Mount Palomar Observatory. For a statistical investigation, however, only about 2/3 of the clusters listed in the Catalogue have been used, and those that could be observed on moderatly low galactic latitudes, i.e. under poorer observing conditions, have been excluded. On the basis of the Catalogue, analogous of Table 4a and Figure 1d related to the equatorial region of the Galaxy have been composed.

Table 9

The distribution of clusters of galaxies according to their distance and richness, as may be deduced from Abell's data for clusters located at low galactic latitudes

Richness Group \ Distance Group	1	2	3	4	5	6
I	2 100%	2 100%	2 66.7%	8 80%	49 77.8%	88 73.9%
II	—	—	1 33.3%	2 20%	11 17.4%	24 20.2%
III	—	—	—	—	1 1.6%	7 5.9%
IV	—	—	—	—	2 3.2%	—
V	—	—	—	—	—	—

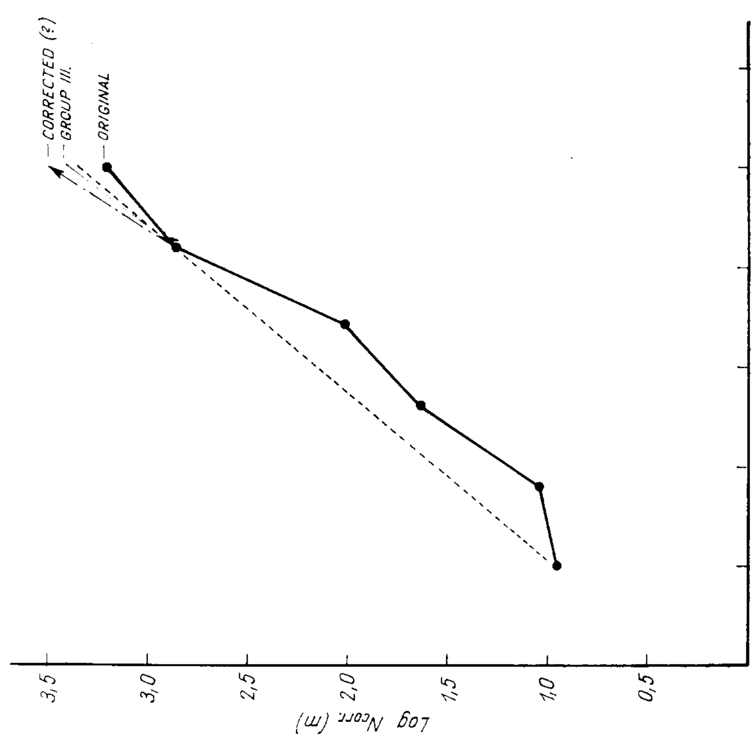

Fig. 5. The functions Log N_A(m) and Log N_{corr}(m)

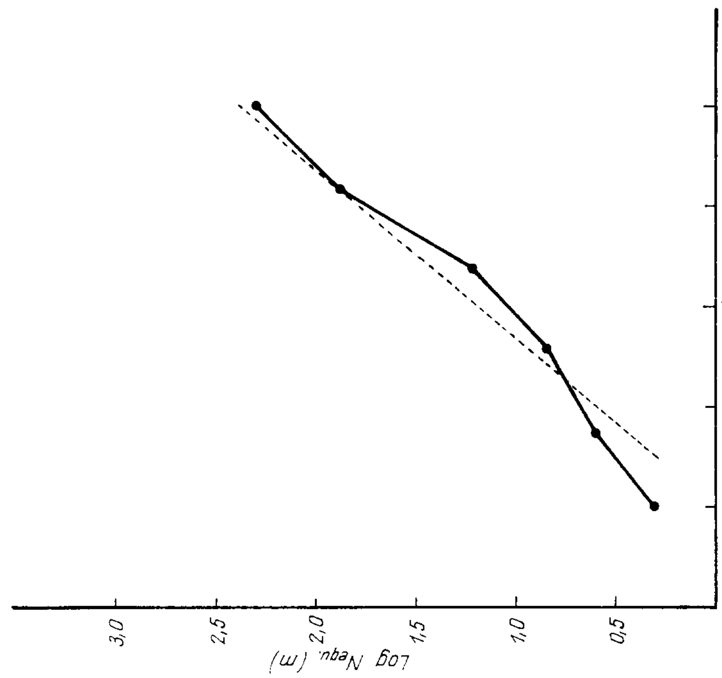

Fig. 6. The Log N_{equ}(m) diagram of Abell's clusters of galaxies at low galactic latitudes

As seen in Figure 6 and Table 9, the correlation between the distance and the richness of the clusters, and the higher steepness of the diagram for richness group III in distance groups 5 and 6, manifests itself clearly also in a poorer, but independent statistical sample.

Putting Figures 2b and 2c of Section 4 side by side it is seen that the counts of medium compact and open clusters in Zwicky's catalogue are strongly reduced by some observational selection. Abell [8a] analyses Zwicky's counting procedure in detail and demonstrates that with this counting procedure in two clusters, of the same type but of different distances, a lower count will necessarily result for the more distant one. In case of clusters of lower central density, this loss proves to be important, whereas in case of compact clusters it plays only a minor role. Following from this it is obvious that the actual diagram Log $N_c(m)$ is slightly steeper than that in Figure 2a obtained by direct observation. This means that even from Zwicky's completely independent data, the slope of the correct Log N curve at the right end exceeds that of the line $N = 0.6$ m + const.

9) *The lowest sections of Log N curves constructed on the basis of Abell's Catalogue suggest the presence of the Local Supergalaxy.*

All Log N functions deduced from Abell's material are characterized by strikingly high N values for the first distance group. Considering the low number of clusters found in distance group 1 and 2, this fact is of no great importance for a statistical investigation. It is worth to note, however, that as shown by independent investigations [14a], field galaxies brighter than 15 magnitudes are also more frequent in space than less bright ones, that is to say, our surroundings to a given distance is likely to form a distinct unit of higher density. Considering statistical uncertainty and possible changes in density in the Local Supercluster there is no need to try to identify the lowest part of the observed Log N curve with that of some homogeneous and isotropic cosmological model.

Notes on the evaluation of the empirical relationships $\vartheta(\delta)$ and $N(\delta)$

Current cosmological theories predict various relations between the quantities m, δ, N and ϑ* principially accessible for observation. The checking of these relations by observation permit to reject inadequate theories [15]. From among all the possible functional relations of the four observable quantities, three are independent. So far only the relationship $m(\delta)$ has been determined to an accuracy suitable for cosmological evaluation [9b]. Our results permit the presumption that the relationships $N(\delta)$ and $\vartheta(\delta)$ also can be determined to a greater accuracy, and thus all relationships of cosmological importance will be cleared up. For the function $N(\delta)$ this has been rendered possible by the elimination of a systematic error, and for $\vartheta(\delta)$ by the application of a new method. As regards further systematic errors in the material

* The function $\vartheta(\delta)$ discussed in Section 6 is identical with the angular diameter — red shift relation required in cosmology, provided only that the average sizes of the clusters of galaxies are unchanged.

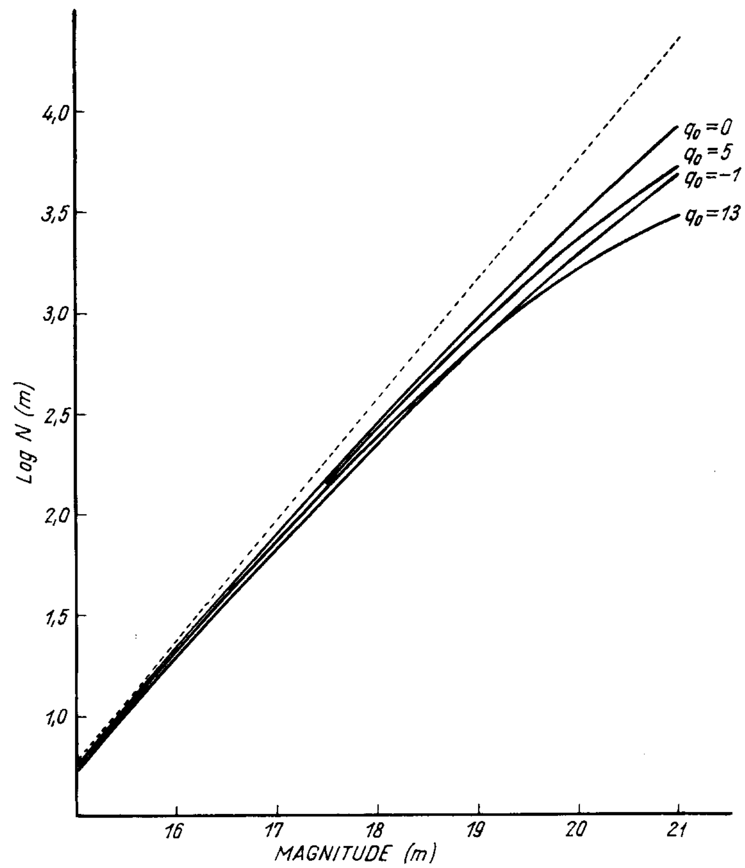

Fig. 7. Theoretical Log N(m) diagrams for steady-state and relativistic cosmological models ($\Lambda = p = 0$).

q_0 = parameter of cosmological deceleration [10c],
Λ = cosmical constant,
p = pressure
(After Sandage [10d])

used no definite statement can be made yet. Consequently all that will be said in connection with the evaluation of the observational relationships. should be considered as tentative.

I. Abell's original Log N_A(m) and Log $N_A(\delta)$ diagrams within the limits of the accuracy of observations fit well with all curves predicted by various current homogeneous cosmological models, whereas the course of the corrected Log N_{corr}(m) diagram considerably differs from any of these curves.

For a comparison, the theoretical Log N(m) diagrams of some of the most important cosmological models are represented here; these are the curves to fit in with the observational points of Figure 5 after corresponding shifts parallel to the axes (Figure 7).

By collating Figure 3 and Figures 13, 14 from Lit. No. [9] it appears that the anomalous increase of the function $N_{corr}(m)$ corrected for the error in the angular diameter does not come from an error in Abell's magnitude scale (from which an error in the opposite sense could be expected) but it is a reflection of some natural phenomenon. In this case the analysis of the correction process tending to eliminate the anomaly observed in the abundance ratios of the various types of clusters would indicate that the difference between empirical and theoretical Log N(m) functions is due to the change of the space frequency of clusters, and not to the simultaneous spreading of their members.

II. If the average sizes of the clusters of galaxies did not undergo significant changes 10^8-10^9 years ago, i.e. if ϑ represents a „metric angular diameter" [10b], then, according to the predictions of the cosmological models investigated by Sandage [10], the Abell's convention regarding the angular measures shows a relative error between distance groups 5 and 6 greater than 15 per cent, only in case when the cosmological deceleration parameter, q_0 is greater than 13. (Based on Table 1 and 6 in Sandage's work referred to as [10].) The determination of the actual error in the angular diameter of the counting circle is still to be expected from further observations, yet the current value $q_0 < 5$ [11b], appears rather unlikely.*

In fair agreement with the above conclusion, theoretical curves of those cosmological models can be fitted best to the right-hand side upper section** of the empirical relationship $m(\delta)$ for which q_0 is of the order of magnitude of 10 at least [10]. It is the more remarkable, that when observations relating to smaller distances and correspondingly nearer times are considered, the best fitting theoretical curve corresponds to $q_0 \approx 2.5$ [9b, 11b]. Possible explanations are an inaccuracy of magnitude measurements, an uncertainty of knowledge regarding the development of galaxies, a lack of information on intergalactic obscuration, the uncertainty of the K-correction, the possible inhomogeneity of the velocity field of the Metagalaxy, a rapid decrease of the deceleration of universal expansion or the anomalous curvature of space. The latter two possibilities require the investigation of more general models of the Metagalaxy than those considered by Sandage.

In a paper to follow it will be shown, considerably more generalized (and partly independently from the Einstein equations) that in case the functions $N(\delta)$ and $\vartheta(\delta)$ are correct the density of the space distribution of large clusters of galaxies goes growing in every direction with the distance. On the other hand, the deceleration parameter, q_0 as determined from data related to distance groups 3, 4, 5 and 6 is not of an order of magnitude of 10 only if the structure of the space differs from that assumed at the deduction of Friedmann's equation, or else if the average sizes of the clusters of galaxies in different distance groups are strongly different. These conclusions apply to models in which the velocity field is homogeneous and isotropic.

As a possible interpretation of the two conclusions it should be mentioned that a metagalactic system, of roughly spherical shape and of a radially increasing density (just as „association-like" clusters of galaxies, often of similar

* In this case the relative error in the radius of the counting circle would be smaller than 10 per cent.

** Observations of distance groups 3—6. Data refer to 10^8-10^9 years back in time.

structures [14b], planetary nebulae and some stellar associations) might be considered as the product of an explosion. On the other hand, the strong deceleration of universal expansion may be the consequence of the gravitational effect of an exceptionally high density of material or of radiation, or else of some law of gravity, different from Newton's law.* It is important to know that when taking such high gravitational effects into consideration the dynamics of the individual large clusters of galaxies and that of the Metagalaxy itself cannot be considered independently [12b, 16–19].

In principle it is possible and in a strongly inhomogeneous model of the Metagalaxy it cannot even be considered unlikely that, contrary to our premise, the field of velocity in the range under examination shows significant systematic inhomogeneity, which means that the changes of velocity resulting from the relationships $m(\delta)$ and $\vartheta(\delta)$ occur not in time (negative acceleration) but in space.** In this case, of course, the dynamical inferences regarding density and law of gravity would not apply.

Finally, we wish to point out once more that the empirical support of the ideas outlined above is not complete yet, and that the definitive clearing up of the problems brought up will need control observations and further discussions.

It is a pleasure to thank my wife for several useful conversations and for extensive help with the numerical work in this paper. Special thanks are due to Dr. I. Almár, for reading and commenting on the manuscript.

* In the general theory of relativity a corresponding modification of the law of gravity is obtained in the case of $\Lambda \ll 0$, where Λ is the cosmical constant [11c].

** The train of ideas developed in the Appendix applies, with slight modifications, also in this case.

APPENDIX

*On the theoretically expected error in Abell's convention regarding the angular diameters of clusters**

The absolute luminosity, L and the apparent luminosity, l, of galaxies are interconnected by the known relationship [20]

$$l = \frac{L}{(1+\delta)^2 \cdot \sigma^2(\omega) \cdot R_0^2}, \tag{1A}$$

where δ is the red shift, $\sigma(\omega) = \begin{cases} \sin\omega \\ \omega \\ \mathrm{sh}\,\omega \end{cases}$ depending on the structure of space, $R_0 = R(t_0)$, $R(t)$ the cosmological scale factor, t_0 the instant of the observation. Using the formulae $\mathrm{m} = -2.5 \,\mathrm{Log}\, l$ and $\mathrm{M} = -2.5 \,\mathrm{Log}\, L$ to pass from luminosities to magnitudes, for apparent and absolute magnitudes the following expression results:

$$\mathrm{m} = \mathrm{M} - 5\,\mathrm{Log}[(1+\delta)\cdot\sigma(\omega)\cdot R_0]. \tag{2A}$$

Starting from the usual metrics of relativistic cosmology, the following relationship between the apparent and actual diameters of an object may be inferred [20]:

$$\Delta = R\cdot\sigma(\omega)\cdot\vartheta, \tag{3A}$$

where Δ is the linear diameter of the object, and ϑ is the angular diameter.

Considering that $\dfrac{R_0}{R} = 1+\delta$, and in Abell's observations $\vartheta = \dfrac{K}{\delta}$:

$$\sigma(\omega) = \frac{(1+\delta)\cdot\delta\cdot\Delta}{K\cdot R_0}. \tag{4A}$$

Substitute the term found for $\sigma(\omega)$ into relationship (2A):

$$0.2\,\mathrm{m} - \mathrm{Log}\,[(1+\delta)^2\,\delta] = \mathrm{Log}\,\Delta + \mathrm{M} + \mathrm{Log}\,K. \tag{5A}$$

Formula (5A) does not include the term $\sigma(\omega)$ which means that it is obtained without the integration of Einstein's equations or of Friedmann's equation, and is independent from them.

Denote the term on the left-hand side of equation (5A) by μ. If the absolute brightness of the galaxies under observation is the same, then

$$\mu = 0.2\,\mathrm{m} - 2\,\mathrm{Log}\,(1+\delta) - \mathrm{Log}\,\delta = \mathrm{Log}\,\Delta + \mathrm{const}. \tag{6A}$$

* This supplement contains the proof of the statement in Section 5 note d.

and

$$2.3 \frac{d\mu}{d \operatorname{Log} \delta} = \frac{1}{\Delta} \cdot \frac{d \Delta}{d \operatorname{Log} \delta}. \tag{7A}$$

Observations made by Humason, Mayall and Sandage [9b] have revealed the following relationship between photovisual magnitudes corrected for red shift spectrum change (K-correction) and the red shift itself*:

Table A_1

Relation between the photovisual magnitudes of the synthetically brightest members of 18 clusters of galaxies and their red shifts

Cluster Number	Cluster Designation		δ	m
1	Virgo		.004	8.27
2	Perseus	0316+4121	.018	11.72
3	Coma	1257+2812	.022	11.80
4	Hercules	1603+1755	.036	13.09
5		2308+0720	.043	13.79
6		2322+1425	.044	14.18
7		1145+5559	.052	14.70
8		0106−1536	.053	14.45
9		1024+1039	.065	14.89
10		1239+1852	.072	14.19
11	Cor. Bor.	1520+2754	.072	14.96
12		0705+3506	.078	15.46
13	Boötes	1431+3146	.131	16.21
14		1055+5702	.134	16.22
15		0025+2223	.159	16.28
16		0128+1840	.173	16.49
17		0925+2044	.192	16.41
18	Hydra	0855+0321	.202	16.70

From Table A_1 the corresponding curve $\mu(\delta)$ may be derived (Fig. A_1). Figure A_1 shows the absolute value of the derivative $\frac{d\mu}{d \operatorname{Log}\delta}$ and, together with it, the relative error in Abell's angular diameters, $\frac{1}{\Delta} \frac{d\Delta}{d \operatorname{Log} \delta}$, to be in general negligibly small for the first four distance groups** (in the same range the abundance ratio of more diffuse clusters does not drop either), whereas from the fifth distance group on it abruptly starts growing. Substituting the derivatives by the change ratios taken from Figure A_1 for the

* The results of the more recent photoelectric observations made by Baum [21] have not been considered here because he had used apertures inversely proportinal to the red shift.

** The derivative of the function $\mu(\operatorname{Log} \delta)$ also in the range $-1.7 < \delta < -1.3$ differs from zero. This is likely to be the result of an excess speed or an excess space curvature occuring in the Local Supergalaxy and might be brought into connection with the anomaly in the abundance ratio of less significance, discussed in Section 2.

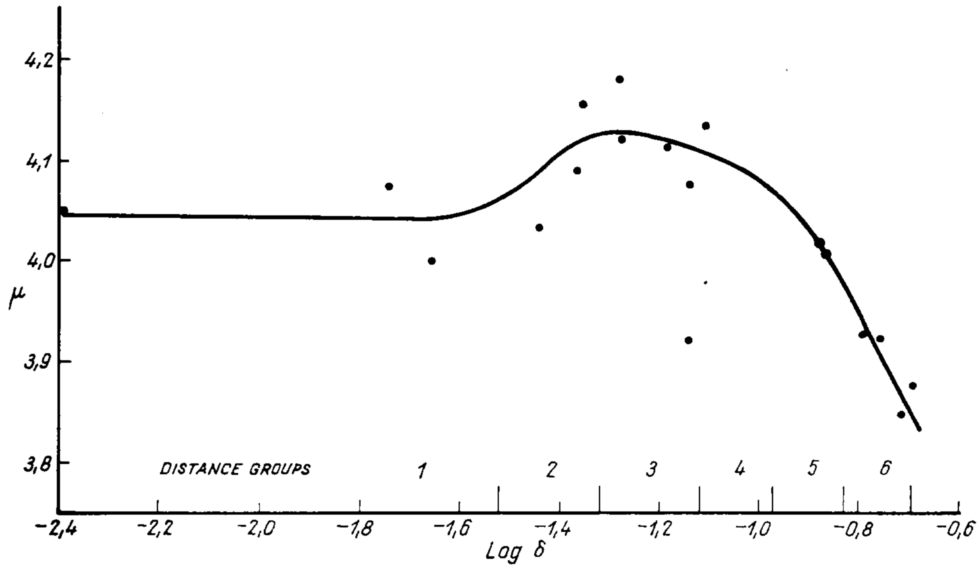

Fig. A_1. $\mu = 0.2$ m — Log $(1 + \delta)^2$. δ versus red shift, for 18 clusters of galaxies

corresponding distances in distance groups 5 and 6 the following equation is obtained

$$2.3(\mu_5 - \mu_6) \approx \frac{\Delta_5 - \Delta_6}{\Delta_5} \approx 0.25. \qquad (8A)$$

The relative error in the angular diameter to be expected between distance groups 5 and 6 appears to be considerably larger than 10—15 per cent. as mentioned by Abell (cf. p. 19). Yet this value is uncertain, for several reasons. Magnitude measurements may be affected by systematic errors. Magnitudes listed in Table A_1 are corrected by the K-term but its value is uncertain. No reliable data on intergalactic obscuration are available and its effect cannot be taken into account. Finally, the scanty information we have on the evolution of galaxies cannot give sufficient support to the assumption used in Formula (6A) according to which M is independent from time. The conspciuous drop of the curve A_1, however, would not correspond to facts unless a sudden systematic drop of about $1^m\!.5$ occurred in the absolute intrinsic brightness of galaxies. It is seen that our conclusion hardly depends on possible cosmogonical changes.

Figure A_1 contradicts the opinion generally spread in astronomy that the red shift is a linear function of „distance", which is also the starting point of Abell's working hypothesis. Observations indicate that „distance by apparent size" [11c] and the concept of distance based on red shift from distance groups 4 on are sharply different [10f].

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A kiadásért felelős Detre László. – Műszaki szerk. Tóth Erzsébet
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64.58687 Akadémiai Nyomda, Budapest — Felelős vezető: Bernát György