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THE ROLE OF PHOTONEUTRINOS IN THE EVOLUTION OF THE STARS

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by

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During the stellar evolution, after being exhausted a certain type of thermonuclear reaction, a gravitional contraction follows, usually with an increase of the central temperature. Arriving at the temperature 3—5 · 10^9 °K all nuclear energy sources are exhausted, the star becomes a victim of a supernova explosion. A cooling of the centre, however, can be guaranted by means of the transformation of the thermal radiation into neutrinos, so the supernova fate can be prevented for a wide class of stars. Here the photoneutrino-pair transmutation in a plasma (in the Coulomb-field of nuclei) is treated. The influence of the degeneracy of electron gas is discussed separately.

1. Production of neutrinos in thermonuclear reactions

The energy output of stars is usually due to thermonuclear reactions. If at a certain temperature all the isotopes needed for the possible reaction-type are exhausted, the star, which was formerly in a thermodynamical equilibrium, starts a gravitional contraction, which produces the rising of the central temperature. The contraction procedure goes futher, until the central temperature reaches the threshold temperature of an other thermonuclear reaction. Then it starts a new period, with a steady density and temperature distribution, which lasts until exhausting the isotope reserve of the new reaction type.

In the temperature interval 1.4—2 · 10^7 °K, with 10—100 g cm^{-3} central density, the only reaction is the H → He fusion. (This is the energy source of the main sequence stars.) At the temperature 10^8 °K, with 10^4—10^5 g cm^{-3} densities (red giants) the He → C fusion starts. About 10^9 °K, 10^6 g cm^{-3} the different channels of the C → Fe fusion series play the main role. Above 3 · 10^9 °K, with 10^7 g cm^{-3} most of the channels are open. The chemical elements are in a statistical equilibrium, the most frequent element is the Fe, having the lowest energy content pro nucleon. At this point the energy production of the thermonuclear reactions ceases, the energy radiation, however, still lasts, therefore a catastrophical supernova explosion occurs.

The relatively low abundance of the supernovas shows, however, that not all of the stars finish their evolution with a supernova explosion. This means, that in the majority of cases, the central temperature does not reach the critical value 3 · 10^9 °K. The reason why the uprising of the central temperature stops under this critical value is one of the fundamental problems of stellar evolution. It may be, that this cannot be explained by means of usual thermodynamical and mechanical principles and by the facts of the conventional nuclear physics. In the last years a new possibility was discussed: at high temperatures the neutrino radiation of the star increases, which may serve, thus, as a cooling mechanism, necessary for explaining the observed low supernova abundancy.

Neutrinos can be emitted only in weak interactions. Neutrino creation was observed until now only in spontaneous decay processes. At the threshold temperature of the first thermonuclear reactions (10^7-18^8 °K) the β decays following the fusion processes, result a neutrino output. In the radiation of the main sequence stars, the percentage of the neutrino energy is 2—10% [1]. The neutrino power of the spontaneous decay processes, however, is not sufficiently high to play a decisive role in the stellar development. On the other hand, the time of the collision in a plasma is too short compared to the characteristic time of weak interactions. In very big stars (at very high temperature and density) the situation can be changed. Although the cross section of the weak reactions is lower with many orders of magnitude than the cross section of nuclear and electromagnetic reactions, the main free path of the neutrinos is far more longer than that of electrons, photons, neutrons or ions, longer than the diameter of the stars. The number of the neutrinos produced in a hot plasma is less than the number of other particles, all the neutrinos, however, get out of the star while the other particles mainly get caught in the thick stellar matter. Thus one can quite well imagine that under certain conditions the neutrino production by collision may play an important role in the stellar energetics. (If this turns out to be true one can say that the weak interactions become important in stellar dimensions as in the case of the even weaker gravitational interaction.)

In 1941 Gamow and Schönberg [2] draw the attention first to the possibility of neutrino production by collision: the URCA process produces neutrinos by nuclear electron capture, followed by a β decay, as

$$e^- + (Z) \rightarrow (Z-1) + \nu, \quad (Z-1) \rightarrow (Z) + e^- + \tilde{\nu}. \tag{1}$$

The URCA circle diminishes the kinetic energy of the electron gas. This process, however, has a threshold energy, just the energy difference of the isobars $(Z-1)$ and (Z). Therefore at low temperatures only a few and rare groups of isotopes can take part in the URCA process [3]. At high temperatures saturation occurs: here the URCA process is not negligible, but does not play a dominant role.

It was mentioned first by Pontecorvo [4] that by assuming a direct weak $(e\nu)(e\nu)$ coupling on theoretical grounds there are numerous other possibilities for producing neutrinos from the kinetic energy content of the hot electron gas. E. g.

$$e + Z \rightarrow Z + e + \nu + \bar{\nu} \qquad \text{(Bremsstrahlung-neutrinos [5])}, \tag{2}$$

$$\gamma + e \rightarrow e + \nu + \bar{\nu} \qquad \text{(Compton-neutrinos [6])}, \tag{3}$$

$$\left.\begin{array}{l} \gamma + Z \rightarrow Z + e^- + e^+ \\ e^- + e^+ \rightarrow \nu + \nu \end{array}\right\} \qquad \text{(Annihilation neutrinos [6, 7, 8])}. \tag{4}$$

The general features of these processes are that they have no threshold energy and they are combined to electromagnetic interactions (Fig. 1), which diminishes slightly the probabilities of the reactions. The cross-section of these processes compared to the corresponding pure electromagnetic reactions are smaller with 20—25 orders of magnitude. The most important reactions are (3) and (4) at the densities and temperatures occurring in stars. The electron-positron gas being in statistical equilibrium with the heat radiation can annihilate with a small probability to $\nu - \nu$ pairs. The neutrino radiation gets out from the stars without any absorbtion, thus in great scale it produces a considerable cooling. Above 10^9 °K the energy loss reaches the value 10^{15}—10^{20} erg g^{-1} sec^{-1}, which may compensate the heat input coming from gravitational contraction [7].

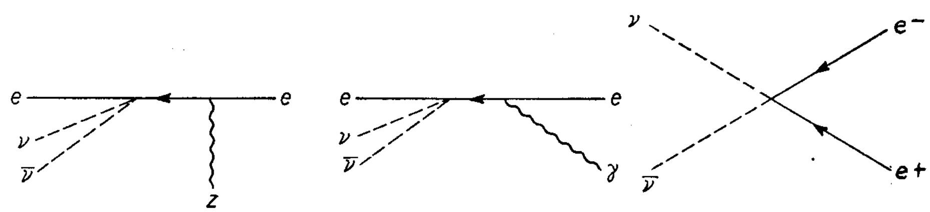

Fig. 1

The situation considerably changes in a degenerated electron gas. In this case the electron states are filled up to a rather high Fermi level, thus the $e^- - e^+$ pair creation below the Fermi level is forbidden by the exclusion principle. Therefore the neutrino production can be diminished with many orders of magnitude. The cross-section of the Bremsstrahlung and Compton neutrinos also diminishes because of the occupied electron states. In a highly degenerated plasma those processes play the most important role, where in the final states real electrons do not appear. These are the photoneutrino-processes:

$$\gamma + \gamma \to \nu + \bar{\nu}, \tag{5}$$

$$\gamma + \gamma \to \gamma + \nu + \bar{\nu}, \tag{6}$$

$$\gamma + Z \to Z + \nu + \bar{\nu}. \tag{7}$$

Here the high energy γ photons of the thermal radiation produce neutrinos via virtual electron pairs (fig. 2, 3). The cross-sections are also here influenced by the degeneracy of the electron gas, but — presumably — not so sensitively as in the case of real electrons. The cross-section of the reaction (5) vanishes for symmetry reasons [9] if the $(e\nu)$ $(e\nu)$ coupling is a local one. Calculation for the cross-section of the process (6) are in course [10]. The

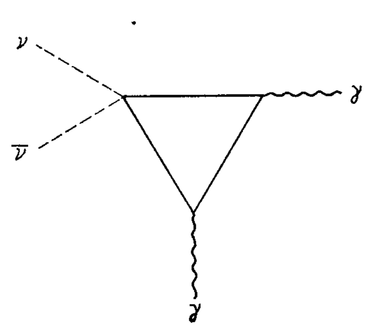

Fig. 2

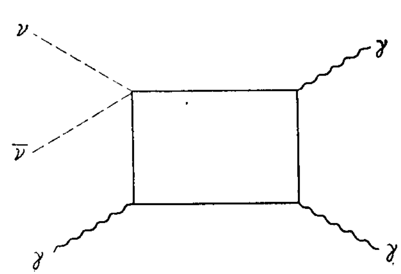

Fig. 3

cross-section of the process (7) is presumably higher because of the presence of the factor Z^2. Here the process (7) will be treated in details.

2. *The cross-section of photoneutrino production*

A local electron-electron coupling is supposed in the form

$$H(x) = \frac{f}{\sqrt{2}} [\overline{\Psi}_\nu \gamma_\mu (1 + \gamma_5) \Psi_e] [\overline{\Psi}_e \gamma_\mu (1 + \gamma_5) \Psi_\nu] .$$

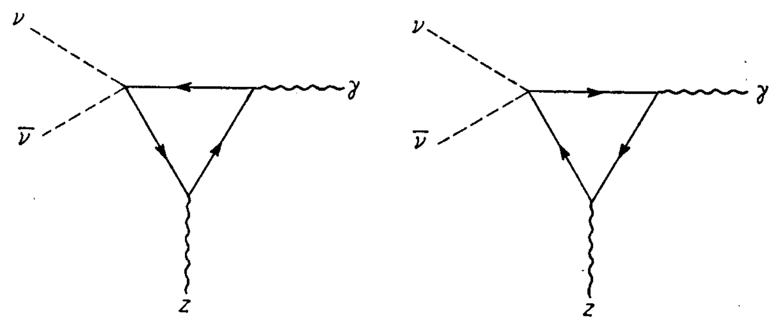

Fig. 4

This is equivalent — according to the Fiertz identity — to the interaction

$$H(x) = \frac{f}{\sqrt{2}} [\overline{\Psi}_e \gamma_\mu (1 + \gamma_5) \Psi_e] [\overline{\Psi}_\nu \gamma_\mu (1 + \gamma_5) \Psi_\nu] .$$

The Hamiltonian of the electromagnetic interaction is, of course,

$$H'(x) = ie \overline{\Psi}_e \gamma_\mu A_\mu \Psi_e .$$

The first non-vanishing contribution comes from the diagrams plotted in Fig. 4, which correspond to the process

$$\gamma + Z \to Z + (e^- + e^+)_{\rm virt} \to Z + \nu + \bar{\nu} .$$

The S matrix element is

$$S = \frac{(-i)^3}{(2\pi)^3} \frac{Z e^3 f}{V^{3/2} \sqrt{k_{10}}} \bar{u}_\nu \gamma_\alpha (1+\gamma_5) v_\nu \cdot$$

$$\cdot e_\beta n_\gamma \cdot \int \frac{M_{\alpha\beta\gamma}(k_1, k_2)}{|\vec{k}_z^2| + D^{-2} \hat{\lambda}_C^{-2}} \delta(k_{20}) \delta^4(k_1 + k_2 - k_\nu - k_{\bar{\nu}}) d^4 k_2 .$$

Here k_1 and k_2 are the momenta of the real incoming photon and of the virtual photon representing the Coulomb-field, e_β is the polarization vector of the incoming photon, n_γ is the unit vector, representing the direction of the time axis. The potential of the Coulomb-field is $A_\gamma = i n_\gamma Z e r^{-1} \cdot \exp (-r/D\hat{\lambda}_c)$. $2\pi\hat{\lambda}_c$ is the electron Compton wave length, $D\hat{\lambda}_c$ is the screening parameter, characteristic to the state of the plasma. (In the case of a bare nucleus $D = \infty$.) According to the classical theory of the plasma screening,

$$D^{-2} = \frac{(4\pi)^2}{137} \frac{Z}{A} \frac{\varrho/\varrho_0}{kT/m_e c^2}, \tag{8}$$

where

$$\varrho_0 = M \lambda_c^{-3} = 4.37 \cdot 10^7 \text{ g cm}^{-3}.$$

The matrix element $M_{\alpha\beta\gamma}$ represents the contribution of the closed electron loop:

$$M_{\alpha\beta\gamma}(k_1, k_2) = \operatorname{Tr} \int \gamma_5 \gamma_\alpha S_c(p + k_1) \gamma_\beta S_c(p) \gamma_\gamma S_c(p - k_2) d^4 p .$$

Here, in the explicit calculations one obtains some improper integrals, these, however, can be eliminated because of the gauge invariance:

$$M_{\alpha\beta\gamma} k_{1\beta} = 0, \quad M_{\alpha\beta\gamma} k_{2\gamma} = 0. \tag{9}$$

The calculation of the total cross-section can be made more conveniently if the integration is carried first out for the two neutrino momenta leaving the recoil momentum as the integration variable in the last integral. Using the notation

$$k_{10} = E \lambda_c^{-1}, \quad (k_1 \cdot k_2) = \lambda_c^{-2} E^2 x, \quad (k_2 \cdot k_2) = \lambda_c^{-2} E^2 y, \tag{10}$$

the total cross-section is given by

$$\sigma_D(E) = \sigma_0 Z^2 D^4 E^{10} \cdot \tag{11}$$

$$\cdot \int_0^{4} \int_{y/2}^{\sqrt{y}} \frac{A^2(-x^2 y^2 + 4xy^2 - y^3) + 4B^2(-x^4 + x^2 y) + 4AB(-x^3 y + xy^2)}{6(1 + D^2 E^2 y)^2} dx\, dy,$$

where

$$\begin{aligned} A(E, x, y) &= \int_0^1 \int_0^{1-\xi} \frac{\xi - \xi^2 - \xi\eta}{1 + E^2 [y(\xi - \xi^2) + 2x \xi\eta]} d\eta\, d\xi, \\ B(E, x, y) &= \int_0^1 \int_0^{1-\xi} \frac{\xi\eta}{1 + E^2 [y(\xi - \xi^2) + 2x \xi\eta]} d\eta\, d\xi, \end{aligned} \tag{12}$$

and

$$\sigma_0 = \frac{f^2 m_e^2}{137^3 \cdot \pi^4} = 1{,}34 \cdot 10^{-52} \text{cm}^2. \tag{13}$$

Let us take the low energy limit as first. For this case the integration can be carried out easily. Without any screening of the nuclear Coulomb-field we get:

$$\sigma(E) = 0{,}0035\, \sigma_0 Z^2 E^6, \qquad E \ll 1 \tag{14}$$

(the neglected terms are E^2-times smaller). With screening

$$\sigma(E) = 0{,}151\, \sigma_0 Z^2 D^4 E^{10}, \qquad E \ll 1 \tag{15}$$

(with the same neglection). The values for higher energy can be calculated only numerically. We see, however, that the high energy limit $E \gg 1$ gives a quadratical energy dependence of

$$\sigma(E) \simeq 1{,}62\ \sigma_0 Z^2 E^2\ . \tag{16}$$

The main conclusion is that the cross section is a very quickly increasing function of E, reaching the order of magnitude of σ_0 in the MeV region. Only photons with this energy produce photoneutrinos in a considerable extent. More details about the calculation will be given elsewhere [11].

3. *Photoneutrino production in a degenerated gas*

Our results in (14) and (15) are valid only if there are no many electrons present in the reaction domain. In a very dense plasma, however, where the electron gas is degenerated, a coherent contribution will come from the following type of reaction:

$$\gamma + e^- + Z \to Z + e^- + \nu + \bar{\nu}. \tag{17}$$

(17) is allowed by the exclusion principle only if the initial and the final electron states are the same. In this case the electrons of the Fermi sea are the catalizators of the transmutations in the nuclear Coulomb-field on the same way, as in the case of (7) the negativ energy electrons of the Dirac-sea help the same transmutation of heat radiation into the more penetrating neutrino radiation. The matrix element, connecting the initial and final states, in this degenerated electron gas, in the lowest order of perturbation theory, has the following structure:

$$\langle \gamma, z, \mathrm{Gas}\,|S|\,\nu, \bar{\nu}, Z, \mathrm{Gas}\rangle = S_\alpha + \sum_{p_e < p_F} S_\beta(p_e) + \sum_{p_e,\, p_{e'} < p_F} \sum S_\gamma(p_e, p_{e'}), \tag{18}$$

where the S_α, S_β, S_γ terms correspond to the diagrams of the Fig. 5. (Permutations of the corners are not shown separately.)

In the § 2 the matrix element S_α is given. The summarized contribution of all these diagrams can be given most elegantly by a re-definition of the vacuum state. Let us call "ground state" $|\,F >$ the state, in which all the electron states are filled up to a given Fermi momentum p_F. If we introduce the new electron-propagator by the definition

$$S_C^F(x - x') = \langle F \mid T\,\Psi(x)\,\overline{\Psi}(x') \mid F\rangle\ , \tag{19}$$

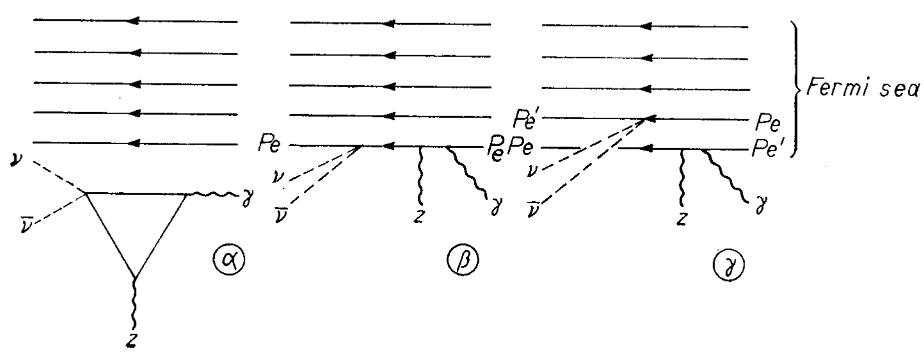

Fig. 5

the whole matrix element (18) can be symbolized by the simple diagram pair of the Fig. 6, where the contributions of the internal electron lines are not the usual causal propagators $S_c(x, x')$, but the modified causal propagators defined through (19). The explicit calculation of the cross-section of photoneutrino production is on this way principally easy, but practically rather complicated, and can be done only for some special cases. A more detailed nformation about this work may be found elsewhere [11], here only the main esults will be quoted.

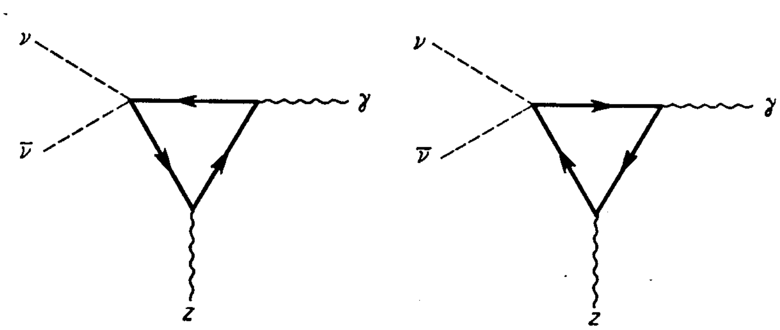

Fig. 6

It is a justified assumption, that in a definit energy region the first term of (18) will dominate. In this case the cross-section can be represented by (11) in a good approximation. In an other domain the first term (the contribution of the Dirac-sea) of (10) can be neglected, only the electrons of the Fermi-sea will have a remarkable contribution to the cross-section. In the intermediate energy region both matrix elements are of the same order of magnitude, here an interference occurs between the diagram α and $\beta + \gamma$. In this energy region the formulas are rather complicated, their explicit evaluation would be a very difficult job. Let us concentrate our efforts on the two asymptotic case. The limits of the "α" region and of the "$\beta + \gamma$" region depend on the value of the Fermi momentum i. e. on the plasma density very sensitively.

The calculation can be simplified by the assumption that p_F (measured in $m_e c$ units) is small. In this case the cross-section of the pure "Fermi-sea reaction" (diagram $\beta + \gamma$) would be

$$\sigma = 1{,}21\,\sigma_0 Z^2 p_F^4 D^4 E^2, \quad \text{if} \quad p_E \ll 1, \quad E \ll p_F, \quad E \ll D^{-1}, \tag{20}$$

$$\sigma = 1{,}65\,\sigma_0 Z^2 p_F^4 D^2 \left[1 + 0\left(\frac{1}{4 D^2 p_F^2}\right)\right], \text{ if } \begin{cases} p_F \ll 1, & 4 p_F^2 D^2 \gg 1, \\ E \gg p_F, & E \gg D^{-1}. \end{cases} \tag{21}$$

The corresponding formulas of the pure "Dirac-sea reaction" (diagram α) are given by (15) and (16).

Let us take a definite example, in which the plasma density is about $\varrho = 10^5\,\mathrm{g\,cm^{-3}}$, the plasma temperature $t = 0{,}59 \cdot 10^8$ °K and $A = 2Z$. Under these conditions

$$p_F = {}^1/_3, \qquad D = 5. \tag{22}$$

Now

$$\sigma = 94{,}5\ \sigma_0\ Z^2\ E^{10}\ \text{(Dirac-sea, } E \ll 1) ,$$

$$\sigma = 9{,}34\ \sigma_0\ Z^2\ E^2\ \text{(Fermi-sea, } E \ll 0{,}33) ,$$

and

$$\sigma = 1{,}62\ \sigma_0\ Z^2\ E^2\ \ \text{(Dirac-sea, } E \gg 1) ,$$

$$\sigma = 4{,}48\ \sigma_0\ Z^2\ \ \ \ \text{(Fermi-sea, } E \gg 1) .$$

Comparing these values it can be calculated that below 1 MeV the Fermi-sea catalyzed reaction (17), at several MeV the Dirac-sea catalyzed reaction (7) dominates. At about 1 MeV the two reactions are competing, the order of magnitude of the cross-section is several times $Z^2\sigma_0$. In the whole energy region as a very rough approximation the functional dependence $\sigma \sim E^2$ can be used. The decisive role of the screening is clear, its neglection is not allowed.

4. *Neutrino radiation of the hot plasma*

From astronomical point of view we are interested in the problem: what is the rate of the (irreversible) transmutation of the thermal electromagnetic radiation into neutrino radiation at a given absolute temperature t in a second in one gram of the plasma material.

The energy distribution of the photons is given by the Planck law. These photons collide to the nuclei (to be considered at rest) and produce neutrino pairs with the total cross section

$$\sigma(E) = \sigma_0 Z^2 F(E), \tag{23}$$

Thus one obtains the neutrino output density (in erg g^{-1} sec^{-1} units)

$$P(T) = \frac{1}{\pi^2}\frac{Z^2}{A}\frac{m}{M}\frac{\sigma_0 c^3}{\lambda_c^3}\int_0^\infty \frac{F(E)\,E^3}{e^{E/T}-1}\,dE = P_0\frac{Z^2}{A}G(T), \tag{24}$$

where

$$P_0 = 3{,}5\cdot 10^6\,\frac{\text{erg}}{\text{g}\cdot\text{sec}} \tag{25}$$

and

$$G(T) = \int_0^\infty \frac{F(E)\,E^3}{e^{E/T}-1}\,dE\,. \tag{26}$$

Here $T = kt/m_e c^2$ is the absolute temperature, expressed in $t = 5{,}9\cdot 10^9$ °K units.

The value of P depends in first approximation only on the plasma temperature T, on its chemical composition (charge number Z and mass number A), and is independent of its density.* In Fig. 7 the Planck distribu-

* This is true only for very thin gases. In the realistic case the cross-section depends through the screening length $D\lambda_c$ on the temperature and on the electron density in the plasma, and this causes a modification in the ϱ- and T-dependence of P.

tion, the energy dependence of the cross-section and their product is plotted. It can be seen, that photons of a not very wide energy domain (at about $E = 5T$ or $7T$) play the most important role in the photoneutrino generation.

In our special example $\varrho = 10^5$ g cm^{-3}, $t = 53 \cdot 10^6$ °K, $Z = 26$ (iron plasma) one can use the approximative formula

$$F(E) = C(E)E^2,$$

where $C(E)$ is a slowly varying function somewhere between 1 and 10, so we arrive at the numerical result $P \sim 5 \cdot 10^{-3}$ erg/g sec.

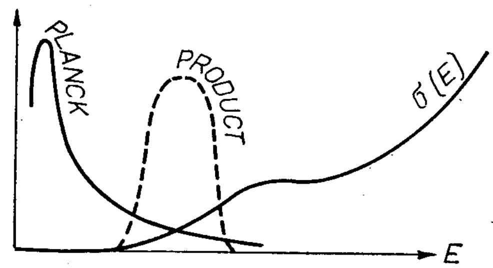

Fig. 7

The extrapolation for higher temperatures and densities is a rather difficult job. The density sensitivity of $\sigma(E)$ and $P(T)$

$$n_\varrho = \frac{d \log \sigma}{d \log \varrho} = \frac{d \log P}{d \log \varrho}$$

comes from the screening parameter D and for small energies $n_\varrho \sim -{}^2/_3$, for higher energies it becomes zero or positive at low densities, but the density dependence is not very strong. The temperature sensitivity of the cross-section

$$n_T = \frac{d \log \sigma}{d \log T}$$

is something between 0 and 2, thus the temperature sensitivity of the neutrino power density

$$\bar{n}_T = \frac{d \log P}{d \log T}$$

varies between 5 and 8 in the case of low degeneracy. At about $t = 10^9$ °K and higher density the power density may reach the value $P = 10^6$ erg/g sec.

It is instructive to compare our results to the intensity of the other neutrino producing processes. The difficulty is, that the value of P depends for the reaction (7) only through D but in the case of the processes (1)—(4) also explicitely on the plasma density. This dependence will be very important and very complicated in a degenerated electron gas. Calculations for this case are not completed for all types of reactions. But comparing the P values given by different authors, one can conclude, that well above

10^8 °K the annihilation neutrinos (coming from the reaction (4)) will be dominating. Below 10^9 °K the Compton-neutrinos (produced through the reaction (3)) give the most important contribution. In a degenerated plasma, however, the photoneutrinos (coming mainly from the electron-catalyzed reaction (17)) will have a compating intensity at lower temperatures. Just this region is very important for understanding the development of stars having intermediate masses. It is not excluded that in the case of our Sun just the photoneutrinos will prevent the supernovas explosion. For a definite answer one needs the knowledge of the function $P(T, \varrho)$ more accurately, especially for the case of degeneracy for all reaction types.

For showing the cooling role of the neutrinos let us define the cooling time τ as a ratio of the energy content of the electromagnetic radiation in unite volume of the plasma (given by the Stefan—Boltzmann law $u = a\,T^4$) and the neutrino energy power in unite volume ($P\varrho$):

$$\tau = \frac{a\,T^4}{P\varrho}.$$

For a plasma with high density (about 10^7 g/cm^3) and high temperature (about 10^9 °K) it will be smaller, than a month. This shows that the cooling mechanism of neutrinos will be very rapid, compared to astronomical time intervals, if the star approaches the values $T \sim 1$, $\varrho \sim \varrho_0$.

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