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Nr. 47

S. M. KUNG (Nanking)

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by

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Abstract: 33 results including 3 results of this paper in connection with the zero point of the period-luminosity relation since Wilson's result of 1939 are tabulated. They are collected in 6 tables according both the nature of the results and the methods used. A comment for each result is made. 11 results are rejected for various reasons stated in section II. The absolute magnitude of cepheids fainter more than 1^m in Wilson's result, according which the original zero point of Shapley's P—L relation was fixed is fully explained. The expression for the correction due to the spread both of the apparent and absolute magnitudes of the stars is derived in section III. With the correction thus derived the correct formula for the product of the mean distance and the mean parallax, $\bar{r} \cdot \bar{\pi}$, is obtained. Finally, in section IV all the useful results are collected separately to get 3 weighted means, one for the difference between the observed magnitude and the calculated magnitude, based on the assumed absolute magnitude of the RR Lyrae variables and the Shapley's P—L relation, one for the correction to the zero point of the P—L relation, and one for the absolute magnitude of the RR Lyrae variables. With the double weight for the first one, we obtain from all the 3 kinds of data taken together the correction to the zero point of the Shapley's P—L relation to be $-1^m\!.28$ and the absolute magnitude of the RR Lyrae variables to be $+0^m\!.34$.

I. Introduction

Once the period-luminosity relation (P—L relation hereafter) was pointed out by Miss Leavitt in 1908, the importance of this relation was soon noticed by various astronomers. They started to determine the absolute magnitude of the cepheids so as to fix the zero point of the P—L relation. Almost all the cepheids are so distant from us that the method of the trigonometric parallax can hardly be applied to them. The absolute magnitudes used to be derived by the method of mean parallax from their proper motions and radial velocities. This method requires accurate data of the proper motions and radial velocities for a large number of cepheids. Unfortunately, the requirement can not be satisfactorily fulfilled and eventually the zero point of the P—L relation has to be determined anew whenever there are published new data of the proper motions and radial velocities of cepheids. The early results are mentioned in Shapley's Star Cluster [1]. The so-called zero point correction raised in the recent years is referred to the Shapley's P—L relation of 1940 [2].

$$\dot{M}_{pg} = -0^m.28 - 1^m.74 \text{ Log } P. \qquad (1)$$

In equation (1), P is the period in days; $\dot{M}_{pg}$ the median photographic absolute

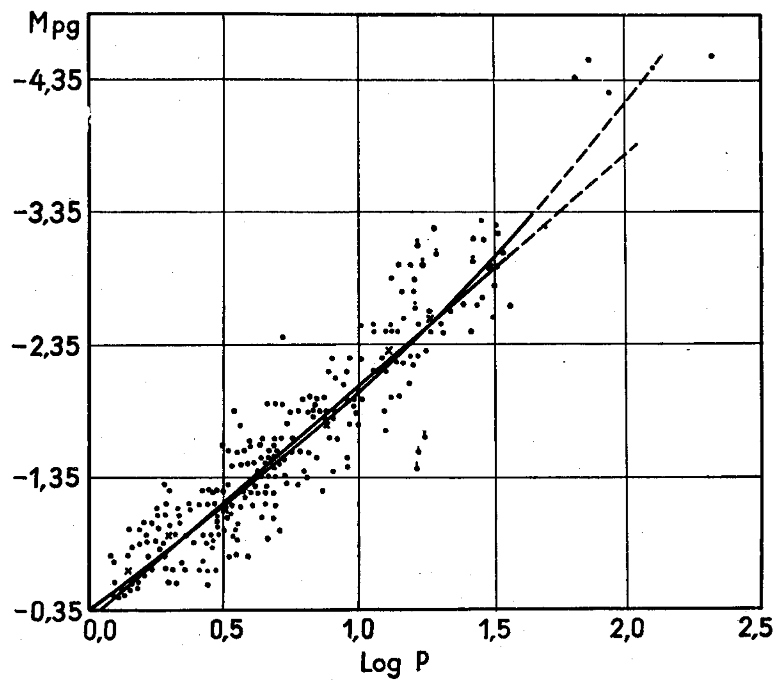

Fig. 1.

magnitude which is equal to $1/2(M_{max} + M_{min})_{pg}$. For convenience we will write M_{pg} for $\dot{M}_{pg}$ henceforth.

The coefficient in equation (1) was obtained from 307 cepheids in the Small Magellanic Cloud (SMC hereafter) by plotting the apparent magnitudes and Log P. The constant term fixing the value of the zero point was mainly based upon Wilson's results [3] which will be discussed in detail in section II. For demonstrating the extent of the scattering of the star points in the m-Log P plotting, Shapley's diagram is reproduced here as figure 1. He pointed out that in the range of $+0.08 < \text{Log } P < 1.6$, equation (1) represented the observational data satisfactorily, but the curve in figure 1 has the best representation.

Several works related to the zero point of the P—L relation have been done in the Soviet Union [4] [5] [6].

The real challenge to the zero point of the P—L relation was raised by Baade at the eighth general assembly of I. A. U. in 1952 [7]. He pointed out that the zero point should be about $1^m.5$ brighter. Actually his observational data only indicates the discrepancy of $1^{m}.5$ between the observed apparent magnitude of RR Lyrae variables and the value to be excepted from Shapley's P—L relation with the assumed photographic magnitude of RR Lyrae variables, $\overline{M}_{pg} = 0^{m}.0$. He opinioned that the absolute magnitude of RR Lyrae variables, $\overline{M}_{pg} = 0^{m}.0$, was correct and the discrepancy was wholly due to the zero point of the P—L relation. His reason is as follows: both the absolute magnitude of the cepheids and the RR Lyrae variables are mainly determined by the mean parallax method from their proper motions. The cepheids are situated close to the galactic plane where the effect of interstellar absorption is considerable and the proper motions of cepheids are small and so are less reliable. Moreover, he considered that the results of color magnitude diagram of M3 obtained by Sandage gave the evidence, $\overline{M}_{pg} = 0^{m}.0$. Later he still held this point in his speech for the zero point problem [8], although he emphasized too the importance of the determination of the absolute magnitudes both for the cepheids and the RR Lyrae variables.

Whether this discrepancy of $1^{m}.5$ is wholly contributed by the error of the zero point of the P—L relation needs careful examination. Sandage later stated elsewhere [9] that the most reliable value of the absolute magnitude of RR Lyrae stars was obtained by the mean parallax method and enumerated four points to explain why the color magnitude diagram of M3 was unable to fix the absolute magnitude of the RR Lyrae variables. In the past few years the photoelectric observations of stellar magnitudes in two or three colors have provided more accurate data for the interstellar absorption. And the large number of the proper motion and radial velocity data for the cepheids is always favourable to the determination of the absolute magnitude of the cepheids by the statistical mean parallax method.

There have been, so far as we know, about 30 results in connection with the zero point of the P—L relation since Wilson's work of 1939. Some old results are no less useful if some corrections are applied and the new data of interstellar absorption are used instead. Some latter ones which either were obtained by unjustified simple approximations, incorrect formulas or by inadequate use of theoretical equations have to be rejected. Results obtained by different methods should be given proper weights in taking the average. Moreover, results which make use of both the P—L relation and some assumed absolute magnitudes of the RR Lyrae variables should be collected separately in one group. In the light of this latter method, the results obtained for the absolute magnitude of the RR Lyrae stars have to be included and collected in a separate group too in order to obtain the final result. It is therefore worthwhile to examine all these results carefully and to make some corrections if necessary and finally to obtain a weighted mean value of all useful results on a reasonable basis.

II. The Various Results of the Correction to the Zero Point of the P—L Relation and of the Absolute Magnitude of the RR Lyrae Variables

The P—L relation of the cepheids is at present mainly an empirical relation. The rather large scattering of the points in figure 1 means that it is only a statistical relation. Therefore, in weighting each result, the number of stars used is essential as to get the correction to the zero point of the P—L relation by the method of trigonometric parallax, the mean parallax from proper motions or radial velocities or any method of utilizing individual stars.

Table 1 contains 4 results of the difference between the observed and calculated magnitudes. All of them are obtained by the assumption that the photographic absolute magnitude of the RR Lyrae stars, $\overline{M}_{pg} = 0^m\!.0$. A brief account of each method is stated in table 1. With the exception of Weaver's method [10], the other three methods are straightforward. Once M_{pg} is determined, the correction to the zero point readily results after a simple subtraction. All these values of difference are very important, because the methods are so direct that there is no other error introduced besides the small values of interstellar absorption and the uncertainties of the determination of the stellar apparent magnitude. In Baade's method, since both the cepheids and the RR Lyrae variables are situated in the Andromeda nebula, the effect of interstellar absorption can be neglected. Of course, it is assumed that the cepheids in it are of the same type as those in the SMC. Similarly, in Shapley's two methods, [11] [12] the globular clusters in the Milky Way are assumed to be the same as those in SMC and LMC; in Weaver's method the cepheids in our Galaxy are assumed to be the same as in the SMC. According to the other characteristics, such as light curves and spectra, the above assumptions are valid.[1]

Weaver's method is rather indirect that the correction to the zero point of the P—L relation is obtained through the determination of the distance to the galactic center, R_0. He put the 21 cepheids he used into two groups and then each group results in a single curve in the plane of R_0 and ω_0', the derivative of the angular velocity of the Galaxy with respect to the distance to the galactic center. The probable error of this method will manifest itself more clearly when each curve is drawn for every single star as that in the determination of the longitude and the latitude by the method of an astrolabe. Thus, the weight for this method is less than the other three.

Parenago [6], utilizing the average apparent magnitudes of 36 cluster type variables of 3 globular clusters in LMC and SMC, $\overline{m}_{pg} = 18^m\!.6$, determined by Thackeray and Wesselink [13] and the absolute magnitude of the cluster type variables, $\overline{M}_{pg} = +\,0^m\!.5$, determined by Pavlovskaya [4], obtained their modulus $m - M = 18^m\!.1$. As the zero point of the Kukarkin's relation was obtained on the basis of the modulus equal to $17^m\!.8$, he derived the correction, $-\,0^m\!.3$, applied to Kukarkin's P—L relation. Parenago did not take into account the effect of interstellar absorption. If the value of the interstellar absorption, $0^m\!.5$, as given by Thackeray and Wesselink, and $\overline{M}_{pg} = 0^m\!.0$ are taken, then the correction applied to Shapley's P—L relation will be $-\,0^m\!.8$. This result while not included in table 1, may be worthy of reference.

Table 2 tabulates 9 results of the correction to the zero point of Shapley's P—L relation. They are obtained by the method of the mean parallax from proper motions. In table 2, V_0 is the velocity of the solar motion relative to the cepheids; A_0, D_0, the right ascension and declination of the

[1] Kron and Svolopoulos, however, pointed out recently the discrepancies in color excesses between the galactic and SMC cepheids. (P. A. S. P. 1959, *71*, 126.)

Table 1

The Difference, $\Delta' M_{pg}$, between the Observed and the Computed Magnitudes (O—C). The computed magnitudes are based on Shapley's P—L relation and the absolute magnitude of the RR Lyrae variables, $\overline{M_{pg}} = 0^m\!.0$

Authors	Brief Account of the Methods	Int. Abs.	M_{pg}	Wt
Baade[1] 1952	The apparent magnitudes of the brightest stars of Population II in M31 are observed. The absolute magnitudes of them are known to be $1^m\!.5$ brighter than that of the cluster type variables. The effect of the interstellar absorption (Int. Abs.) is almost the same for the cepheids and these brighest stars and can be neglected	Considered	(p. e.) $-1^m\!.5$	2
Shapley[2] 1953	The average total magnitude of a globular cluster is derived from 13 clusters in LMC and SMC and is compared with that derived from 30 galactic globular clusters. The average absolute magnitude of the latter is deduced from their cluster type variables	$0^m\!.3$	$-1^m\!.3 \pm 0^m\!.2$	2
Shapley[3] 1954	The apparent magnitudes of the 6th and 30th brightest stars are determined in each of 10 globular clusters of LMC and SMC. Their respective magnitudes thus derived are compared with the apparent magnitudes of the 6th and 30th brightest stars in each of 13 galactic globular clusters. The absolute magnitudes of the latter are deduced from the cluster type variables in their respective clusters	$0^m\!.4$	$-1^m\!.6 \pm 0^m\!.2$	2
Weaver[4] 1954	The distance R_0 to the galactic center is determined through the effect of the galactic rotation on the radial velocity of the cepheids. From 21 cepheids, $R_0 = 3.8$ kpc, is obtained, on the basis of Shapley's P—L relation to derive the distance of the cepheids. Baade obtained $R_0 = 8.7$ kpc by other method on the assumption, $\overline{M_{pg}} = 0^m\!.0$. In order to agree with the Baade's result, the correction to the zero point of the P—L relation is derived	Considered	$-1^m\!.43 \pm 0^m\!.30$	1

(1) W. Baade, 1954, I. A. U. Trans. 8, 397.

(2) H. Shapley, 1953, Proc. Nat. Acad. Sci. 39, 349.

(3) H. Shapley and V. M. Nail, 1954, Pro. Nat. Acad. Sci. 40, 1.

(4) H. Weaver, 1954, A. J. 59, 375.

* The value given by Weaver is $-1^m\!.56$. Later Baade obtained a better value of $R_0 = 8.2$ kpc. According to Weaver's remark that the correction to the zero point will be modified by $\pm 0^m\!.13$ if the value of R_0 has an error of $\pm 0^m\!.5$ kpc. hence the value, $\Delta' M_{pg}$, is corrected accordingly.

Table 2

The Correction, ΔM_{pg}, to the Zero Point of Shapley's P—L Relation. Derived from the method of the mean parallax with the proper motion data.

Authors	No. of Stars	V_0 km/sec	A_0	D_0	l_0	θ km/sec	Corr. Spread m	Corr. Spread M	Int. Abs.	$\bar{\pi}$ $1'' \times 10^{-3}$	$\overline{m_{pg}}$	$\overline{M_{pg}}$	$\overline{\text{Log } P}$	ΔM_{pg}	Wt.
Wilson[1] 1939	86	28.1	270°	+36°	326°	14	No	$-0^m.07$	$0^m.63$ $0^m.85$/kpc	1.35	$+7^m.96$	-2.09	+0.89	$-0^m.26$[1] $\pm 0^m.22$	
Mineur[2] 1944	85	30.2	280°	+29.5	337°	Developing distance into series. See section II.			0.62/kpc	$\lambda = 2.58$, $\mu = -1.06$				$-1^m.77$ $\pm 0^m.32$	
Berthod-Zaborowski[3] 1946		29.8							$1^m.64$/kpc	$\lambda = 0.77$, $\mu = -0.11$.				$-0^m.10$ $\pm 0^m.12$	
Kukarkin[4] 1949	86	20.0	270°	+36°	326°	14	No	$-0^m.09$	$1^m.21$ A (r. l. b.)	1.73	$+8^m.04$	$-2^m.06$	+0.89	$-0^m.23$[2] $\pm 0^m.22$	
Blaauw & Morgan[5] 1954	18	21.0	270°	+30°			Considered		$0^m.7$	3.07[3] 2,72				$-1^m.57$[4] $\pm 0^m.3$	
Parenago[6] 1954	18	18.6	270°	+30°			Considered		$0^m.7$					$-1^m.47$[5] $\pm 0^m.3$	1.0
Gascoigne & Eggen[7] 1957	17						Considered		$A_{pg} = 4\,E$					$-1^m.72$[6]	
This Paper	86	20.0	270°	+36°	326°	14	$-0^m.90$		$2^m.13$ $1^m.58$/kpc	1.73	$+8^m.07$	$-3^m.77$	+0.89	$-1^m.94$[7] $\pm 0^m.32$	1.5
This Paper	86	20.0	270°	+36°	326	14	$-0^m.90$		$1^m.21$ A (r, l, b).	1.73	+8.07	$-2^m.85$	+0.89	$-1^m.02$[8] $\pm 0^m.22$	

(1) R. E. Wilson, 1939, Ap. J. 89, 218.
(2) H. Mineur, 1944, Ann. d'Astroph. 7, 160.
(3) H. Berthod—Zaborowski 1946, Ann. d'Astroph. 9, 123.
(4) B. V. Kukarkin, 1949, Variable Stars, USSR 7, 69.
(5) A. Blaauw and H. R. Morgan, 1954, B. A. N. 12, 95
(6) P. P. Parenago, Variable Stars, USSR 10, 193.
(7) S. C. B. Gascoigne and O. F. Eggen, M. N. 117, 430

[1] The value ΔM_{pg}, being $-0^m.38$ given by Wilson, should be $-0^m.39$. Wilson refferred to the old P—L relation the zero point of which has a difference of $0^m.13$ with that reffered now.

[2] The value is derived on the basis of Wilson's result.

[3] $3''.07 \times 10^{-3}$, $2''.72 \times 10^{-3}$ are the new values of the mean parallax derived respectively from those of Joy and Parenago.

[4] The value is $-1^m.4$ given by Blaauw and Morgan. Eggen etc. pointed out it to be $0^m.17$ too faint,

[5] The value is obtained by adopting $V_0 = 18.6$ km/sec on the basis of the Blaauw and Morgan's result.

[6] The value is derived by using the proper motion data given by Blaauw and Morgan, excluding one star, W gem.

[7] The value is obtained by applying corrections to Wilson's result.

[8] The value is obtained by applying correction to Kukarin's result.

solar apex; θ, the mean peculiar motion of the cepheids. The other notations are self-explanatory. These results are discussed one by one as follows:

The interstellar absorption of $0^m.85$/kpc adopted by Wilson [3] is definitely too low as compared to recent values. In his method to get the mean parallax, the correction for the spread of the apparent and absolute magnitudes is necessary as will be stated in section III. Wilson did not apply the correction for the spread of the apparent magnitudes. It amounts to $0^m.8$ as shown in the next section. For these reasons, Wilson's result can not be adopted. His result, however, can still be used after some corrections are applied as those of this paper are obtained on the basis of Wilson's results.

Mineur [15] treated the cepheids and the RR Lyrae variables separately in his calculation, but he put them together in the last step to get the correction to the zero point and the value of interstellar absorption. Now we clearly know that the cepheids belong to the population I with well-known P—L relation and the RR Lyrae variables belong to the population II with very low correlation between their periods and luminosities.* They should be not mixed together. We, therefore, compute the corrections to the zero point and the values of interstellar absorption separately for the cepheids and the RR Lyrae variables according to the method and the data given by Mineur. The results so obtained are included in tables 2, 3 and 4. Then to our surprise, we get the absurd result: the interstellar absorption for the cepheids is about $0^m.63$/kpc and that for the RR Lyrae variables is $1^m.13$/kpc. This contradictory result, we find out, arises from the method Mineur employed.

Mineur's method may be stated briefly as follows: Let r be the distance derived on the basis of taking the interstellar absorption, a/kpc, and the absolute magnitude M obtained from some P—L relation; r', the corresponding distance derived from $a + \delta$ and $M + \Delta$. Then it results,

$$m - M = 5(\mathrm{Log}\, r + 1) + ar$$

$$m - M - \Delta = 5(\mathrm{Log}\, r' + 1) + (a + \delta)\, r'$$

and so one will get

$$5\, \mathrm{Log}\, (r'/r) = -\Delta - r'\,\delta - a(r' - r) \qquad \text{(M1)}$$

Assume

$$r' = \lambda\, r + \mu\, r^2; \quad \lambda,\ \mu,\ \text{two constans} \qquad \text{(M2)}$$

Substituting (M2) into (M1), one obtains

$$5\, \mathrm{Log}\, \lambda + 5 \times 0.434\ \mu\, r/\lambda = -\Delta - \{\lambda\, \delta + a(\lambda - 1)\}\, r \qquad \text{(M3)}$$

then

$$\Delta = -\mathrm{Log}\, \lambda, \quad \delta = -5 \times 0.434\ \mu/\lambda^2 - a(\lambda - 1)/\lambda. \qquad \text{(M4)}$$

In starting the computation, the zero point of the P—L relation and the value of interstellar absorption, a, are assumed, then Δ is the correction to

* From 668 RR Lyrae variables in 16 globular clusters, Kukarkin [4] derived in the range $-0.356 < \mathrm{Log}\, P < -0.225$ the following P—L relation for them:

$$M_{pg} = -0^m.58 - 0^m.203\, \mathrm{Log\, P}, \quad \text{for Log P} < 0$$

the zero point and δ is the correction to a. In the actual computation, he wrote $1/r' = 1/(\lambda r) - \mu/\lambda^2$ and substituted it in the equations of the proper components to get λ and μ.

The advantage of Mineur's methods is that one needs not pay any attention to the correction for the spread of the apparent and absolute magnitudes of stars. However, his many assumptions and approximations are questionable. First, two terms of r to represent r' in equation (M2) is not adequate. Secondly, equation (M3) is obtained after making the approximation $\mathrm{Log}_e(1 + \mu r/\lambda) = \mu r/\lambda$ and neglecting the two terms, $\mu r^2 \delta$ and $\mu a r^2$. As in his case, $\mu/\lambda = 0.3 \sim 0.4$, $a = 0^m\!.7$/kpc, $|\mu| = 0.5 \sim 1.0$ and the value of r may attain 4, considerable errors will be introduced by the above procedures. Lastly, the expression for $1/r'$ is too simple to be reliable. Thus Mineur's result has to be rejected.

Herthod—Zaborowski [16] employed the same method as that of Mineur's and her result was obtained on the basis of the latter. By the same reason, we will not adopt this result.

Kukarkin [4] applied several corrections to Wilson's result. He made use of the Parenago's absorption formula [17], $A = a_0 A(b, r)$ and the values of $a_0(l, b)$, l, b being the galactic longitude and latitude and so obtained the correction to the interstellar absorption to be $1^m\!.21 - 0^m\!.63 = 0^m\!.58$. There are some small modifications for the average apparent magnitude and for the correction to the spread of the absolute magnitude. A rather large correction of $+0^m\!.55$ comes from adopting $V_0 = 20.0$ km/sec instead of Wilson's 28 km/sec. The reason for this is because Wilson did not exclude the distant stars for which the Oort formula of the galactic rotation does not apply well. However, Kukarkin also did not make the correction for the spread of the apparent magnitude and thus his result cannot be adopted either.

Blaauw and Morgan [18] selected 18 cepheids with large and accurate values of proper motions. They determined the mean parallax of these 18 cepheids by utilizing the parallax values of Joy and Parenago, reducing the proper motion components to those at the same parallax and determining the differences of the mean parallax with those of Joy's and Parenago's. By doing so, it needs no correction for the spread of the apparent and absolute magnitudes of the cepheids. The parallax values of Joy and Parenago are derived respectively from the P—L relations of Shapley and Kukarkin. Hence the corrections to the zero point of these two P—L relations are obtained. Blaauw and Morgan took the average interstellar absorption to be $0^m\!.7$ based on Eggen's result [19] of the interstellar reddening of the 9 cepheids they used. This value, $0^m\!.7$, is approximately median between Joy's $0^m\!.38$ and Parenago's $1^m\!.35$. The correction to the zero point thus obtained was $-1^m\!.4$. Due to the adopted apparent magnitude being too faint by $0^m\!.17$ as pointed out by Eggen et al. [20], the correction hence is now $-1^m\!.57$ in table 2.

Parenago [6] made one modification for Blaauw and Morgan's result. The latter took $V_0 = 21.0$ km/sec. Parenago adopted $V_0 = 18{,}6$ km/sec, in consideration of the effect of negative K term. He calculated the corrections to the zero point for both Kukarkin's old and new P—L relation and obtained the average value $-0^m\!.8$. Applying the correction of $-0^m\!.17$ pointed out by Eggen et al. and reducing to Shapley's P—L relation, we obtain the correction to the zero point to be $-1^m\!.47$ as listed in table 2.

Gascoigne and Eggen [21] also employed the Blaauw and Morgan's proper motion data of 17 cepheids to determine the zero point of the P—L relation, excluding W Gem for its lack of photometric data. From their photoelectric data they derived the intrinsic color of the cepheids to be $+0^m.25$. The interstellar absorption for each cepheid was determined individually by observing its color and applying the relation, the total photographic absorption $A_{pg} = 4\,E$, E being the color excess. The average absorption thus obtained is $1^m.58$/kpc. All the proper motion components were reduced to those at the same distance of 300 kpc, so no correction is needed for the spread of the apparent and absolute magnitudes. No values of V_0, A_0 and D_0 were mentioned. Their correction to the zero point is $-1^m.72$.

In the last two rows of table 2 are the results of this paper. The first one is obtained by applying corrections to Wilson's result. First, we adopted Kukarkin's value of $V_0 = 20.0$ km/sec instead of Wilson's 28,1 km/sec. The different values of V_0 will affect the parallax π_g, obtained from the proper motion components parallel to the solar motion and not the parallax π_r, obtained from the components perpendicular to the solar motion. After taking the weights 3 and 1 respectively for the former and the latter as Wilson did, we obtain the resulting parallax $\bar{\pi} = 1''.73 \times 10^{-3}$. Next, according to the result of section III, $\bar{\pi} \cdot \bar{r} = 2.342$, instead of Wilson's $\bar{\pi} \cdot \bar{r} = 1$ (inferred from his value of absorption used), it results $r = 1.35$ kpc. Adopting the average interstellar absorption of $1^m.58$/kpc* given by Gascoigne and Eggen above, we obtain the average absorption to be $2^m.13$ for the cepheids used. Lastly, the correction, $-0^m.90$, for the spread of the apparent and absolute magnitudes of the cepheids derived in the next section, is applied. In addition, we find the median photographic magnitude $\overline{m_{pg}}$ of 86 cepheids from the General Catalogue of Variable Stars of 1958 [22] and obtain $\overline{m_{pg}} = 8^m.07$, $0^m.11$ fainter than Wilson's. After taking into account all these corrections, the final correction to the zero point is $-1^m.94$.

The second result of this paper is obtained by applying the correction, $-0^m.90$, for the spread of the apparent and absolute magnitudes of the cepheids to Kukarkin's result. The latter took into account only the correction $-0^m.09$, for the spread of the absolute magnitudes of the cepheids. Besides, our $\overline{m_{pg}}$ is $0^m.03$ fainter than his. Finally, the correction, $-1^m.02$, to the zero point is obtained. Both these two results come from the same origin, from Wilson's result. As the difference of $0^m.92$ between them arises almost wholly from the difference of the values of interstellar absorption used, the need of more accurate photoelectric measurement on interstellar absorption cannot be overestimated.

When we take weighting factors for the last five results in table 2, we regard the first three as one result since they arise from one source and by the same reason the last two results as one too. Realizing the fact that the former employs much less number of stars but with more accurate data of proper motions, we give the weights 1 and 1.5 respectively for the former and the latter.

* This value is very close to the value $1^m.50$/kpc, we derived from color excess data and the relation, $A_{pg} = 3.5$ E, for 184 cepheids given by Walraven, Muller and Oosterhoff. (B. A. N. 1958, *14*, 81.)

Table 3 contains 6 results for the correction to the zero point of the P—L relation, derived from radial velocities and the effect of galactic rotation through the determination either of the mean parallax or of the distance to the galactic center. In table 3, A is the Oort constant of the galactic rotation, the other quantities have the same meanings as in table 2. These results are discussed in order in the following:

By the method of the determination of the mean parallax from the rotation effect of the Galaxy it is mainly to get the value of $\bar{r} \cdot A$, $\bar{r}$ being the mean distance between the sun and the cepheids. r is obtained only after the value of A is assumed. To get the value of A from the radial velocities of cepheids, one used to adopt the distance derived from the absolute magnitude by some P—L relations, as Joy [23], Gascoigne and Eggen [21], Weaver [24], Stibbs[25] and Ho [26] did. There is little meaning to use the values of A thus derived to get $\bar{r}$. One has to use the values of A derived independently of any P—L relation from objects which have the similar distribution in the Galaxy as the cepheids.

Wilson's result [3], as in table 2, cannot be used because the interstellar absorption adopted, 0^m85/kpc, is too small and no correction for the spread of the apparent magnitudes of the cepheids is made. Those of Mineur's and Berthod—Zaborowski's have to be rejected too because they used the same method as before.

Kukarkin stated briefly how to get the result in his paper [4]. It seems that he employed the 125 cepheids which were at a distance less than 2 kpc from the sun. He obtained $\bar{r}A = 27.2$ km/sec. The value of $A = 19.1$ km/sec, kpc was obtained from those stars which have similar dynamical distribution as the cepheids. With new values of apparent magnitude for the cepheids, of the interstellar absorption and of the correction for the spread of the absolute magnitudes, he obtained the mean absolute magnitude $\overline{M}_{pg} = -2^m52$ for the cepheids of $\overline{\text{Log}\,P} = +0.97$. As before, he made no correction for the spread of the apparent magnitudes of the cepheids. Moreover, as shown in the next section, the expression $\bar{\pi} \cdot \bar{r} = \exp\,(0.46\,\sigma)^2$ he used to get $\bar{\pi}$ has to be improved. Hence his result cannot be adopted either.

Weaver's method [10] of deriving the correction to the zero point has been stated above. His correction, -1^m43, based on the absolute magnitude of the RR Lyrae stars $\overline{M}_{pg} = 0^m0$ has been put into table 1. In his article he also independently derived the distance to the galactic center, $R_0 = 8.7$ kpc, from 13 galactic clusters. From this result he obtained the correction, -1^m56, to the zero point. The advantage of Weaver's method is its independence of the value of A, the Oort constant.

The last row in table 3 is the result of this paper. It is obtained by applying corrections to Wilson's result [3]. Wilson adopted Joy's value of $A = 20.9$ km/sec, kpc. It is not adequate because Joy obtained this result by employing Shapley's P—L relation. In recent years there are two mathematically unbiased determinations [27] of the value of A, independent of any P—L relation. Weaver [28] obtained, $A = 13.2$ km/sec, kpc, from the radial velocities of 187 B stars with their absolute magnitudes derived from their spectral types. In a similar way, Petrie et al. [29] derived $A = 17.7$ km/sec, kpc from the radial velocities of 79 B stars and of their 64 interstellar lines. Since both B stars and the cepheids belong to Population I and have the

Table 3

The Correction, ΔM_{pg}, to the Zero Point of Shapley's P—L Relation.
Derived from the galactic rotation effect and radial velocity data

Authors	No. of Stars	V_0 km/sec	A_0	D_0	l_0	rA km/sec	A km/sec kpc	Corr. Spread		Int. Abs.	$\bar{\pi}$ 1″×10⁻³	$\overline{m_{pg}}$	$\overline{M_{pg}}$	$\overline{\text{Log } P}$	ΔM_{pg}	Wt.
								m	M							
Wilson[1] 1939	157	28.1	270°	+36°	326°	27.4	20.9	No	$1^m.04$	$0^m.85$/kpc	0.88	$+9^m.85$	$-1^m.47$	+0.97	$+0^m.50$[1] $\pm 0^m.28$	
Mineur[2] 1944	142	30.2	259°	+45°	332°		15.5	Developing dist. in series. See sect. II		$0^m.66$/kpc	$\lambda = 1.9$,		$\mu = -0.49$		-1^m1 $\pm 0^m1$	
Berthod-Zaborowski[3] 1946	142	29.8	269°	+45°	339°		15.5			$0^m.89$/kpc	$\lambda = 1.65$		$\mu = -0.38$		$-1^m.85$ $\pm 0^m.15$	
Kukarkin[4] 1949	125					27.2	19.1	No	Considered	A(r. l. b)			$-2^m.52$	+0.97	$-0^m.55$ $\pm 0^m.22$	
Weaver[5] 1954	21	20.1	269°	+23°	325°	Through the determination of the distance R_0 to the galactic center. See section II									$-1^m.56$ $\pm 0^m.30$	1
This paper	157	28.1	270°	+36°	326°	27.4	15.4	$-1^m.52$		$2^m.80$ $1^m.58$/kpc	1.84	$+9^m.97$	$-3^m.03$	+0.97	$-1^m.07$ $\pm 0^m.42$	1.5

(1) R. E. Wilson, 1939, Ap. J. 89, 218.
(2) H. Mineur, 1944, Ann. d'Astroph. 7, 160.
(3) H. Berthod-Zaborowski, 1946, Ann. d'Astroph. 9, 123.
(4) B. V. Kukarkin, 1949, Variable Stars, USSR 7, 69.
(5) H. Weaver, 1954, A. J. 59, 375.

[1] The value ΔM_{pg} given by Wilson is + 0.37. There is a difference of $0^m.13$ between the two P—L relations as mentioned in the note of table 1.

[2] The value obtained by applying corrections to Wilson's result.

characteristics of the flat system, we adopt the mean of the two values, i. e. $A = 15.4$ km/sec, kpc. Thus we obtain $r = 1.77$ kpc. According to the correct formula for $\overline{\pi} \cdot \bar{r}$ derived in section III, we obtain $\overline{\pi} = 1''.84 \times 10^{-3}$. With the interstellar absorption, $a = 1^m.58$/kpc, given by Gascoigne and Eggen [21], the average interstellar absorption for 157 cepheids is $2^m.8$. The correction, given in next section, for the spread of the apparent and absolute magnitudes of 157 cepeids is $-1^m.57$. The mean apparent photographic magnitude of 157 cepheids obtained from the General Catalogue of Variable Stars of 1958 [22] is $\overline{m_{pg}} = +9^m.97$ differing little from Wilson's $+9^m.85$, Finally, we obtain the correction to the zero point of Shapley's P—L relation to be $-1^m.07$.

As to the weighting factors for the two results in table 3, we give 1 and 1.5 respectively for those of Weaver's and this paper. This is because while Weaver's method has the advantage of independence of the Oort constant A, which might introduce some error, he employed only 21 cepheids as against 157 in this paper.

Table 4 lists 4 results for the absolute photographic magnitudes of the RR Lyrae variables, derived from their proper motions. The quantities in table 4 have the same meaning as in other tables. We now make some comments on these results as follows:

Wilson [3] reduced all the proper motion components to those at the mean apparent magnitude of the RR Lyrae variables, $+10^m.5$, hence no correction for the spread of the apparent magnitudes is needed. He put the interstellar absorption for the RR Lyrae variables equal to zero because of their high galactic latitudes. This, as pointed out by Kukarkin [4], is not true. For this reason, Wilson's result is not adopted.

Mineur's result [15] is not adopted too because he treated the RR Lyrae variables with the same method as he did the cepheids.

Kukarkin's result [4] was obtained by applying several corrections to Wilson's result. Kukarkin, making use of Parenago's absorption formula [17] and assuming the absolute magnitude of the RR Lyrae variables, $M_{pg} = 0^m.0$, derived the average interstellar absorption of $0^m.69$ for them. The mean apparent photographic magnitude, $+10^m.94$, of the 54 RR Lyrae stars was obtained from the material of the Sternberg Institute. It is $0^m.44$ fainter than Wilson's. A correction of $-0^m.01$ for the spread of the absolute magnitude of the RR Lyrae variables was added. He finally obtained $\overline{M}_{pg} = -0^m.11$.

Pavlovskaya [14] reduced all the proper motion components to the mean apparent magnitude $+10^m.9$ and so no correction for the spread of the apparent magnitude is needed. She calculated the interstellar absorption individually for each star and reduced them by Parenago's formula [30] in the same way as she did for the apparent magnitudes. Thus, some small correction for the spread of the absorptions of different stars was also taken into account. The $\overline{M_{pg}}$ for the RR Lyrae variables thus obtained is $+0^m.5$. Pavlovskaya's method is more rigorous than Kukarkin's, we therefore, give weights 2 and 1 respectively for the former and the latter.

Wilson [3] also derived $M_{pg} = -0^m.38$ from the radial velocities of 67 RR Lyrae stars and the effect of the galactic rotation. This result, as pointed out by Kukarkin [4], is indeed unreliable. First, Wilson used $A = 20.9$ km/sec, kpc which was derived by Joy [23] from the cepheids, dynamically, totally different from the RR Lyrae variables. Next, in the equation of proper motion

Table 4

The Average Photographic Absolute Magnitude of the RR Lyrae Variables, $\overline{M_{pg}}$ derived from the method of the mean parallax with proper motion data

Authors	No. of Stars	V_0 km/sec	A_0	D_0	l_0	θ km/sec	Corr. Spread		Int. Abs.	$\bar{\pi}$ $1'' \times 10^{-3}$	$\overline{m_{pg}}$	$\overline{M_{pg}}$	Wr.
							m	M					
Wilson[1] 1939	55	119	270°	+36°	326°	72	Considered	0	0	0.85	$+10^{m}.5$	$+0^{m}.15$ $\pm 0^{m}.23$	
Mineur[2] 1944	60	144	290°	+42°	Developing distance into series See section II				$1^{m}.13$/kpc	$\lambda = 1.34, \mu = -0.51$		$-0^{m}.32$ $\pm 0^{m}.04$	
Kukarkin[3] 1949	54	119	270°	+36°	326°	72	Considered	$-0^{m}.01$	$0^{m}.69$	0.85	$+10^{m}.94$	$-0^{m}.11$* $\pm 0^{m}.23$	1
Pavlovskaya[4] 1953	69	134	304°	+43°		67	Considered	$-0^{m}.09$	A(r, l, b)	0.89	$+10^{m}.9$	$+0^{m}.50$ $\pm 0^{m}.12$	2

(1) *E.* Wilson, 1939, Ap. J. 89, 218.
(2) H. Mineur, 1944, Ann. d'Astroph. 7, 160.
(3) B. V. Kukarkin, 1949, Variable Stars, USSR 7, 69.
(4) E. D. Pavlovskaya, 1953, Variable Stars, USSR 9, 349.

* The value is obtained by applying corrections to Wilson's result, excluding one star, RW CrB.

components to determine $\bar{r}A$, the solar apex adopted is incorrect because the motion of RR Lyrae variables, taken as whole, relative to the sun arises from the differential orbital motions between them and the sun, not from the peculiar motion of the sun. For the above reason this result is not adopted and is not included in any table.

Table 5 gives 5 results, derived by methods other than above, of correction to the zero point of Shapley's P—L relation. Brief account of these methods is given in table 5. They are given less weight as compared with those above for the reasons stated in the following:

Kholopov [31] determined the correction to the zero point by utilizing the trigonometric parallaxes of 20 cepheids. He did not mention the value of the interstellar absorption. No data of individual parallax are given. Moreover, the mean error, $+ 1^{m}.8$, given is too large indeed.

Parenago [6] obtained the correction to the zero point by employing 63 parallax values of 29 cepheids. There are several negative parallax values the absolute values of which are considerably larger than the probable error, $\pm 0^{m}.009$, of modern methods of the determination of trigonometric parallax. Moreover, the mean square error, $+ 1^{m}.6$, given, is large indeed.

Paranago derived the correction $- 1^{m}.2$, to the zero point by theoretical method [6]. He used 5 equations (*a*) — (*e*) as written in table 5 to derive the P—L relation. At present we still know very little about the internal structure of giants and supergiants [32]. In our work [33] concerning the distribution in Hertzsprung-Russell diagram of stars of different hydrogen contents, the formula used, taking both the opacity and mean molecular weight into account, applies only to the middle and later part of the main sequence, at most to the late *B* stars. The applicability of equation (*a*) needs careful examination. The constant $c = P\sqrt{\varrho}$ of the relation between the period and the mean density varies with period and the observed values of c may be more than twice as much as calculated ones [34]. It is evident that one can not use a single value of $c = 0.045$. Besides, the reliability of the empirical relations (*d*) and (*e*) and the value for limb darkening all need careful considerations. We, therefore, rather have the feeling that while it is always encouraged to explain theoretically some empirical relations, it is not ready yet at present quantitatively to determine the zero point of the P—L relation. Although this result is not adopted, the theoretical approach carried out by Parenago might be wortly of reference.

Pskovsky [35] derived the correction to the zero point by the determination of the absolute magnitude of cepheids from the method of effective luminosity. His method is worthy of recommendation. Only two stars, δ Cep and η Aql, however, are determined for their absolute magnitude. The P—L relation as stated above is an empirical statistical relation. Even if the absolute magnitudes determined for these two stars are perfectly correct, we do not know whether they fall exactly on the line representing the P—L relation. The zero point is determined only after the accumulation of a great number of such data. This kind of work is at present under investigation by Sandage [36], Arp, [37], and Kraft [38], so far as we know.

Becker's method [39] of obtaining the correction to the zero point by the method of interstellar reddening is also worth while to try. Either the data used are not good enough, or the formula, color excess, C. E. $= 1/4\ A_{pg}$ and

Table 5

The Correction ΔM_{pg} to the Zero Point of Shapley's P—L Relation

Authors	No. of Stars	Methods	Brief Acount of the Procedures	ΔM_{pg}	Wt.
Kholopov[1] 1954	20	Trigonometric parallax	From the trigonometric parallaxes of 20 cepheids is derived the average photographic absolute magnitude $\overline{M_{pg}} = -1^{m}.9$, $\Delta M_{pg} = +0^{m}.1$. It becomes, $\overline{M_{pg}} = -3^{m}.0$, after taking into account the effect of interstellar absorption	$-1^{m}.0 \pm 1^{m}.5$	1
Parenago[2] 1954	29	Trigonometric parallax	The weighted mean parallax of $2''.3 \times 10^{-3}$ is derived from 63 parallax values of 29 cepheids, while that of $3''.7 \times 10^{-3}$ is obtained by using the Kukarkin's P—L relation and taking into consideration of interstellar absorption. The correction $\Delta M_{pg} = 5 \operatorname{Log}(2.3/3.71) = -1^{m}.04$ for the old and $\Delta M_{pg} = -0^{m}.93$ for the present Kukarkin's P—L relations are obtained	$-1^{m}.43 \pm 1^{m}.1$	2
Parenago[2] 1954		Theoretical	By employing 5 relations, the bolometric luminosity $L_b = 10^x \mathfrak{M}^y R^z$, (a), $L_b = R^2 T^4$, (b), $P\sqrt{\varrho} = c$, (c), $\operatorname{Log} T = \operatorname{Log} T_0 - k \operatorname{Log} P$, (d) $\mathfrak{M} = R^3 \varrho$, (e), in which L_b, the mass $\mathfrak{M}$, radius R, density ϱ and temperature T are expressed in units of corresponding solar quantities: and x, y, z, T_0 and k are constants, it results the bolometric magnitude, $M_b = C_1\,(x, y, z, c, T_0) + C_2(x, y, z, K) \log P$	$-1^{m}.2 \pm 0^{m}.6$	
Pskovsky[3] 1957	2	Effective luminosity	The photographic absolute magnitudes of $-2^{m}.7$ and $-3^{m}.67$ respectively for δ Cep and η Aql are derived from equivalent widths of certain spectral lines of these stars, the light curves and color indices both given by Eggen and the luminosities of supergiants and subgiants for $F5-G5$ stars given by Pskovsky	$-1^{m}.3 \pm 0^{m}.4$	1
Becker[4] 1958	11	Interstellar reddening	Based on the apparent magnitudes of these cepheids given by various workers, their color excesses and color indices given by Gascoigne and Eggen, the relation between the total absorption and the color excess, C. E. $= \frac{1}{4} A_{pg}$ and lastly, his formula C. E. $= f(m - M)$, Becker determined their absolute magnitudes. ΔM_{pg}'s thus obtained range from $-0^{m}.3$ to $-3^{m}.4$	$-1^{m}.5 \pm 0\ 25$	2

(1) P. N. Kholopov, 1954, Astr. Cir. USSR No. 148.
(2) P. P. Parenago, 1954, Variable Stars USSR 10, 193.
(3) G. P. Pskovsky 1957, Astr. J. USSR. 34, 19.
(4) W. Becker, 1958, Z. Ap. 44, 26.

2

his own function C · E. $= f(m - M)$ for deriving the absolute magnitude are not reliable and applicable in general, the values of correction to the zero point from individual stars show wide scattering ranging from -0^m3 to -3^m4.

Table 6 includes three results of the determination of the absolute magnitude of the RR Lyrae variables by methods other than that given in Table 4. They are also given less weight for reasons stated below:

Parenago [6] obtained the absolute magnitude of the RR Lyrae variables $\overline{M_{pg}} = -0^m2$, from 19 parallax values of 11 RR Lyrae variables. There are four negative parallax values; the absolute values of two of them are larger than 0.009. The mean error is given as $\pm 1^m5$ which is considerable.

Parenago [40] employed the dynamical method to get $\overline{M}_{pg}$. The method seems rather involved. The data such as the density distribution perpendicular to the galactic plane and the empirical coefficients in his dynamical equation could hardly be expected to be good enough for the present purpose. As to the density $\varrho = (6 \pm 5)\ 10^{-24}$ g/cm^3 in the solar neighbourhood, there is considerable uncertainty.

Kholopov [30] obtained $\overline{M_{pg}} = +3^m4$ from the parallax values of 7 RR Lyrae variables. As he reasoned that the RR Lyrae variables belong to the same type of stars as subdwarfs, he finally adopted $M_{pg} = +2^m0$. Such determination is not rigorous. The mean error $\pm 1^m8$ given, is too large indeed. This result hence is given the least weight.

III. The Correction to the Mean Absolute Magnitude for the Spread of the Apparent and Absolute Magnitudes

1. The Correction to the Mean Absolute Magnitude in the Method with Proper Motion.

Wilson [3] employed Strömberg's formula [41]

$$\overline{M} = m_0 + 5 + 5 \operatorname{Log} \overline{\pi} - 5 \operatorname{Log} C, \tag{2}$$

where

$$\operatorname{Log} C = +\ 0.02\ \overline{(M - \overline{M})^2} \operatorname{Log}_e 10. \tag{3}$$

Equation (3) was obtained on the assumption that the distribution of M was according to the normal error curve about the mean absolute magnitude $\overline{M}$. The correction due to the spread of the apparent magnitudes was not taken into consideration in equation (3) because the apparent magnitudes had already been reduced to a fixed apparent magnitude m_0 in Strömberg's paper, i. e. there is a relation between the parallax π and the reduced parallax

$$\pi_0 = \pi \cdot 10^{0.2(m-m_0)} \tag{4}$$

Generally m_0 is taken to be the mean absolute magnitude; in Strömberg's article, $m_0 = 0$. By doing so, no correction for the spread of the apparent magnitudes is necessary. Wilson did not reduce the apparent magnitudes of the cepheids. His reason was that by doing so it would introduce unbalanced error for the faint stars. This reason is not correct.

Table 6

The Average Photographic Absolute Magnitude of the RR Lyrae Variables, $\overline{M_{pg}}$

Authors	No. of Stars	Methods	Brief Account of the Procedures	$\overline{M_{pg}}$	Wt.
Parenago[1] 1954	11	Trigonometric parallax	The 19 observed palallax values of 11 RR Lyrae variables are compared with the computed ones based on $\overline{M_{pg}} = -0.2$ and Parenago's formula of absorption	$+1^m.7 \pm 1^m.0$	2
Parenago[2] 1954		Dynamical	By using the formula for the density of the Galaxy $4\pi G\varrho = C^2 - 2(A^2 - B^2)$, in which G is the gravitational constant and A, B, the Oort constants of galactic rotation, $C = -\frac{\partial^2\Phi}{\partial z^2}$, Φ the potential energy in the direction z, perpendicular to the galactic plane, $\overline{M_{pg}}$ is derived from the relation $C^2 = 9330 \times 10^{0.4(0.1-\overline{M_{pg}})}$, after adopting $\varrho = (6.0 \pm 5.0) \times 10^{-24}$ g/cm^3	$+0^m.5 \pm 0^m.27$	2
Kholopov[3] 1954	7	Trigonometric parallax and H—R diagram	The value of the mean parallax, $\bar{\pi} = (6'' \pm 3'') \times 10^{-3}$, is derived from the trigonometric parallax of 7 RR Lyrae variables. Thus, it it results, $\overline{M_{pg}} = +3^m.4 + 1^m.8$. With the consideration of the position of the subdwarf in the H—R diagram, Kholopov adopted $\overline{M_{pg}} = +2^m.0$ as he reasoned that the RR Lyrae variables belong to the same type of stars as subdwarfs.	$+2^m.0 \pm 1^m.5$	1

(1) P. P. Parenago, 1954, Variables Stars, USSR 10, 193.
(2) P. P. Parenago, 1954, Astr. J. USSR 31, 5.
(3) P. N. Kholopov, 1954, Astr. Astr. Cir. USSR No. 148.

While the correction for the spread of the apparent magnitudes is solved by the above method, it can also be applied to the final result as follows: The formula for the absolute magnitude

$$M = m + 5 + 5 \operatorname{Log} \pi \tag{5}$$

can be written as

$$\pi = 0.1 \cdot 10^{0.2\,(M-m)} \tag{6}$$

Let

$$M - m = \overline{(M - m)} + \Delta \tag{7}$$

then we have

$$\overline{10^{0.2(M-m)}} = 10^{0.2\overline{(M-m)}}\, \overline{10^{0.2\Delta}} \tag{8}$$

From (6) and (8), it follows,

$$\overline{M} = \overline{m} + 5 \operatorname{Log} \overline{\pi} - 5 \operatorname{Log} \overline{10^{0.2\Delta}} + 5. \tag{9}$$

From equation (7), we know $\Delta = (M - m) - \overline{(M - m)}$. The spread of M arises mainly from stars of different periods; the spread of intrinsic absolute magnitudes at the same period can be neglected as compared to the former. Thus we can write $M = a - 1.74 \operatorname{Log} P$ where a, being a constant, will be cancelled out automatically in the difference Δ. Actually, since the spread of the apparent magnitudes is much larger than that of absolute magnitudes, the correction for the spread in equation (9) is mainly due to the spread of m's.

The first result of this paper in table 2 is obtained by applying corrections to Wilson's result [3]. We obtain $\overline{m}_{pg} = +\,8^m_.07$ and $\overline{(M - m)} = -\,9^m_.63$ for the 86 cepheids according to the method just mentioned and the data given by the General Catalogue of Variable Stars of 1958 [22] The correction term, $-\,5 \operatorname{Log} \overline{10^{0.2\Delta}}$, is computed accordingly. It is equal to $-\,0^m_.90$. We also compute the quantity, $\overline{10^{-0.2}} = 1.511$, needed for the relation $\overline{\pi} \cdot \overline{r}$. If the original data given by Wilson are used, we obtain $\overline{m}_{pg} = +\,7^m_.96$, $\overline{(M - m)} = -\,9^m_.49$, $-\,5 \operatorname{Log} \overline{10^{0.2\Delta}} = -\,0^m_.82$. The correction derived from these values differs only $0^m_.03$ from that just obtained. The second result of our paper is obtained by applying this correction term, $-\,0^m_.90$, to Kukarkin's result, instead of his correction term $-\,0^m_.09$, only for the spread of the absolute magnitudes.

2. The Correction to the Mean Absolute Magnitude in the Method with the Radial Velocities and the Effect of the Galactic Rotation.

In this method one obtains the mean distance $\overline{r}$ from $\overline{r}A$ derived by assuming the value of A. Then the mean parallax $\overline{\pi}$ is derived from $\overline{r} \cdot \overline{\pi}$. Wilson [3] and Kukarkin [4] both used Strömberg's formula [41],

$$\overline{r} \cdot \overline{\pi} = \operatorname{Exp} \frac{0.4\, \overline{(M - \overline{M})^2}}{\mathrm{Mod}^2} \tag{10}$$

In formula (10), the distribution of the absolute magnitude M was assumed according to the normal error curve about their mean $\overline{M}$. Strömberg stated clearly that equation (10) was valid only under the condition that the apparent magnitudes of the stars in question were the same. Therefore, the results derived by equation (10) have to be corrected.

The correct relation of $\overline{\pi} \cdot \overline{r}$ taking the spread of both m and M into account can be derived by the following simple way:
From

$$M = m + 5 - 5 \operatorname{Log} r, \tag{11}$$

we can write,

$$r = 10 \times 10^{-0.2(M-m)} = 10 \times 10^{-0.2\overline{(M-m)}} 10^{-0.2\Delta} \tag{12}$$

Then from equation (6), (7), (8) and (12), we have

$$\overline{r} \cdot \overline{\pi} = \overline{10^{+0.2\Delta}} \cdot \overline{10^{-0.2\Delta}} \tag{13}$$

M can be obtained from (9) and (13), or directly from $\overline{r}$,

$$\overline{M} = \overline{m} + 5 - 5 \operatorname{Log} \overline{r} + 5 \operatorname{Log} \overline{10^{-0.2\Delta}} \tag{14}$$

As the scattering of apparent magnitudes plays a main role in equation (13) results obtained from equations (10) and (13) differ considerably. For example, in table 3, Wilson obtained $\overline{r} \cdot \pi = 1.153$ by equation (10) and we obtained $\overline{r} \cdot \pi = 3.266$ by equation (13).

The result of this paper in table 3 is obtained by applying corrections to Wilson's result [3]. We obtain $\overline{m_{pg}} = +9^m\!.97$, $\overline{(M - m)} = -11^m\!.51$ for the by 157 cepheids according to the method stated above and the data given the General Catalogue of Variable Stars of 1958 [22]. The quantities, $\overline{10^{+0.2\Delta}} = 2.02$, and $\overline{10^{-0.2\Delta}} = 1.67$ are computed accordingly. Hence from equation (13) and $\overline{r} = 1.77$ kpc derived from $\overline{r}A = 27.4$ km/sec and $A = 15.4$ km/sec, kpc, we obtain $\pi = 1''\!.84 \times 10^{-3}$ and the correction for the spread of m's $- 5 \operatorname{Log} \overline{10^{+0.2\Delta}} = -1^m\!.52$.

IV. Determination of the Zero Point of the P—L Relation and the Absolute Magnitude of the RR Lyrae Variables.

The determination of the zero point of the P—L, as we see from above, is related to the absolute magnitude of the RR Lyrae variables, we therefore have to take into account all the three kinds of results, namely, the difference, $\Delta' M_{pg}$, between the observed and the calculated magnitudes given in Table 1, the correction, ΔM_{pg}, to the zero point, given by tables 2, 3, and 5 and finally, the absolute magnitude of the RR Lyrae variables given by tables 2 and 6. We should give the three kinds of results proper weights so as to get the most reliable result. The first kind of the results, the difference, $\Delta' M_{pg}$, between the observed and the calculated magnitudes which is obtained in most straightforward way, should be given a higher weight. According to this principle we handle the above results as follows:

First, we consider the corrections to the zero point given by tables 2, 3 and 5, derived, respectively from the proper motion method, the method of the effect of the galactic rotation and other methods. The method of the effect of the galactic rotation is less reliable than the motion method because of uncertainties in the value of the Oort constant A and the limitation of linear formula used. The least reliable are the results derived form all the other methods, either due to the small number of stars used, or to the non-rigorous methods employed or/and to the considerable uncertainties of the data used. We, therefore, give the weighting factors 4, 2 and 1 respectively for results obtained from tables 2, 3 and 5. Thus the weighted mean is shown in table 7.

Table 7 The Corrections ΔM_{pg}, to the Zero Point Obtained from Tables 2, 3 and 5

Methods	M_{pg}	Weight
Table 2, the proper motion method	$-1^m\!.52 \pm 0^m\!.14$	4
Table 3, the method of the effect of galactic rotation	$-1^m\!.27 \pm 0^m\!.28$	2
Table 5, all the other methods	$-1^m\!.36 \pm 0^m\!.55$	1
Weighted mean........	$-1^m\!.43 \pm 0^m\!.14$	

Next, consider the absolute magnitudes, $\overline{M_{pg}}$, of the RR Lyrae variables. From tables 4 and 6 we obtain $\overline{M_{pg}}$ respectively from the proper motion method and the other methods. For the same reason as above, we give weighting factors 4 and 1 respectively for the results obtained from tables 4 and 6. Hence the weighted mean is shown in table 8.

Table 8. The Absolute Magnitudes, $\overline{M_{pg}}$, of the RR Lyrae Variables Obtained from Tables 4 an 6

Methods	M_{pg}	Weight
Table 4, the proper motion method	$+0^m\!.30 \pm 0^m\!.14$	4
Table 6, All the other methods	$+1^m\!.28 \pm 0.51$	1
Weighted mean..........	$+0.50 \pm 0.13$	

From table 1 we obtain the difference, $\Delta' M_{pg}$, $-1^m\!.46$, between the observed and the calculated magnitudes. If there is no error in the observational data and the methods employed are perfectly correct, we should have the following relation:

$$\Delta' M_{pg} + \overline{M_{pg}} = \Delta M_{pg}, \quad \text{or} \quad \Delta' M_{pg} = \Delta M_{pg} - \overline{M_{pg}}$$

Actually it is not so. In order to render the difference, $\Delta' M_{pg}$, with twice weight as compared to the other two, we put the difference equal to

$$1/3 \left\{ 2\,\Delta' M_{pg} + (\Delta M_{pg} - \overline{M_{pg}}) \right\}.$$

The discrepancy between the latter and the term, $(\Delta M_{pg} - \overline{M_{pg}})$, is then equally distributed to the two terms, ΔM_{pg} and $\overline{M_{pg}}$. Thus all the results taken together into consideration yield the final result as shown in table 9.

Table 9 The Final Result of All the Results Taken Together

The difference between the observed and the calculated magnitudes	$\Delta' M_{pg}$	$-1^m.62 \pm 0^m.07$
The correction to the zero point of Shapley's P—L relation	ΔM_{pg}	$-1^m.28 \pm 0^m.08$
The absolute magnitude of the **RR** Lyrae variables	$\overline{M_{pg}}$	$+0^m.34 \pm 0^m.07$

Therefore, the final P—L relation of the cepheids should have the following form:

$$M_{pg} = -1^m.56 - 1^m.74 \,\mathrm{Log}\, P$$
$$\pm 0.08 \pm 0.06$$

The result obtained above is closely related to the weighting method we have adopted. An attempt is made to do justice to all the results, but it is very likely subject to personal preference or even prejudice. As we have tabulated all the results separately and each result complete with the intermediate data, it is easy for one to get other final results from the tabulated results by using other weighting factors, by making some corrections, or/and by combining with some future new results.

The probable errors shown in tables 7, 8 and 9 are derived from those given by authors in tables 1—6. They should not be attached too much significance, for the errors in interstellar absorption are, in general, not taken into consideration by various authors and are subject to uncertainties, if considered. Moreover, the methods of computation, or more exactly, estimations of errors, are very likely different for different workers. In our computation we give the errors of similar methods to those results which are not given with errors, such as $+0^m.2$ to Baade's result in table 1 and $+0^m.3$ to Gascoigne and Eggen's result in table 2. For those which do not indicate what kind of errors they are, we treat them as propable errors. The average errors and mean square errors given by some authors for their results are all reduced to probable errors so as to make them consistent in tables 1—6.

It is a pleasure to thank Mr. Hsia Ch'ang-li who did most of the numerical computations of this work. The author is indebted to Professor Kukarkin who called his attention in private communication to the work of the southern cepheids by Walraven and his colleagues. He also wish to thank professor L. Detre, the visiting professor at Purple Mountain Observatory during the winter of 1959, for reading this paper and for valuable criticism.

Added in proof: After the present work was completed, three articles related to our work have appeared. 1) The work of Eggen and Sandage (M. N. 119, 255, 1959) on "The Groombridge 1830. group of high velocity stars and its relation to the globular clusters" determined, by the method of moving group parallax, the mean visual absolute magnitude of 7 cluster type variables including RR Lyrae itself to be $M_v = +0^m.6$. Although this value of $M_v = +0^m.6$, as authors stated, might be considered as preliminary due to the lack of accurate photometry for most of the variables, it indicated the same trend with our result that the absolute magnitude of RR Lyrae variables should be fainter than the usually adopted value of $0^m.0$. 2) The absolute magnitudes of cepheids and RR Lyrae stars were discussed by Sidney van den Bergh (J. R. A. S. Canada, 54, 49, 1960) in connection with the extragalactic distance scale. He summarized the recent data of absolute magnitudes of 7 cepheids in galactic clusters which where determined by fitting their respective main sequences with the age zero main sequence in the color—magnitude diagram. He obtained the P—L relation $M = -1.0 - 2.0$ Log P which corresponded $\overline{M} = -2^m.5$ at $\overline{\text{Log P}} = 0.745$. It will give the correction $\Delta M = -0^m.93$ to the zero point of Shapley's P—L relation if the slope -1.74 is used instead of -2.0. He adopted the mean absolute magnitude of RR Lyrae variables $\overline{M} = +0^m.5$ by taking a distance modulus $m - M = 18^m.8$ for the SMC from this P—L relation and the observed median apparent magnitude $\overline{m} = 19^m.3$ of SMC RR Lyrae stars obtained by Thackeray and Wesselink. 3) In a symposium on the differences among globular clusters, Arp. (A. J. 64, 441, 1959) obtained, by fitting the main sequence of clusters to the Hyades age zero main sequence, the absolute magnitudes of RR Lyrae stars, $M_v = +1^m, +0^m.8$, and $+0^m.2$, respectively in $M2$, $M5$ and $M13$. A mean value of $+0^m.65$ is preferred. From the above works it seems that RR Lyrae stars do not form a homogenous group and may have a considerable range in absolute magnitudes, especiallg in different clusters. I am indepted to professor L. Detre for calling my attention to the first two works mentioned here.

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