ON THE MAGNETIC EFFECTS OF TURBULENCE IN IONIZED GASES

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In gases which are good electric conductors (ionized gases) local magnetic fields can rise spontaneously by turbulence. These fields may be produced by the diffusion of electrons due to fluctuating temperature. *Batchelor*'s stability criteria show that highly ionized gases are instable to magnetic disturbances at temperatures above 10 000° C. Based on the analogy between vorticity and magnetic field we may estimate the effects produced in perfect conducting gases, by making use of the vorticity transport hypothesis. We may infer that the specific conductivity of the gas decreases by some orders of magnitude and the magnetic permeability increases. This agrees with the investigations of *Sweet*, who has similarly come to this conclusion. A necessary condition is given by this theory for the origin of the permanent magnetic field of the stars derived from the high ionization and turbulence.

By introducing new functions the equations of the electromagnetic hydrodynamics may be written in symmetrical form analogous to the equations of motions in the classical hydrodynamics. The equations also remain analogous in the case of homogeneous isotropic turbulence.

Introduction

The spectra of many stars show particularly high turbulence in their atmosphere [1]. To explain the physical state of such stars the results of the turbulence theory — obtained in the laboratory — could not be applied directly. Namely the gases in the interior of the stars as well as in their atmosphere are very highly ionized and therefore perfect conductors. The different charges, electrons and ions, are easily displacing each other and thus local magnetic fields can rise instantaneously.

In the presence of external fields the generating of such fields is evident. The external magnetic field exerts forces upon the electrons and ions in a different way, hence the original neutral distribution may undergo a change.

The temperature of a gas in thermodynamical instability may essentially differ in some points from that of the surroundings. But simultaneously with the temperature the rate of ionization changes as well, resulting in the diffusion of electrons in order to equalise the density difference of the free electrons. Therefore, the number of electrons would surpass that of the ions at places of lower temperature, preventing thereby a further diffusion. Lack or prevalence of the electrons would generate local electric fields and in the case of mechanical motions (i. e. eddy motions) magnetic fields as well.

Hence, the condition for producing local magnetic fields instantaneously is given in the thermodynamical and mechanical instability of the gas. The increase or decrease of such an electromagnetic field depends upon the relation between the diffusion of the mechanic and magnetic energy. *Batchelor* examined the stability of the conductive liquids to instantaneous magnetic disturbances. The condition of stability of a liquid in turbulence according to him is as follows :

$$\frac{\varepsilon}{\varrho} \begin{matrix} < \\ = \\ > \end{matrix} \frac{c^2}{4\pi\mu\sigma} \qquad \begin{matrix} \text{stable} \\ \text{indifferent} \\ \text{instable} \end{matrix} \tag{1}$$

where ε/ϱ is the kinematic viscosity, μ the magnetic permeability, σ the specific conductivity (in E. S. U.). In stability the energy of the disturbance will diminish and after a certain time the field disappears. Again in the case of instability the energy would increase exponentially at the beginning. (The increase of the energy may be attributed to the influence of *Alfvén*'s magnetic depression. With the appearence of a disturbing field, the pressure, temperature and rate of ionization diminish, intensifying thereby the diffusion of electrons as well as the electromagnetic field itself.) A limit for the intensifying of a disturbing field is set up by the equilibrium between mechanical and thermodynamical effects on the one hand and the electromagnetic field originated by the shift of the charges, on the other hand.

According to the above stability criteria the Sun's photosphere is stable to magnetic disturbances. The kinematic viscosity coefficient on the solar surface is about 10^{-8}. The magnetic permeability slighty differs from the unit, while the specific conductivity is of the order 10^{-12}, that is, the damping of the magnetic energy will be greater than 10^{-7}.

In considering stars showing instability we find, of course, a completely different situation. Under a constant density the kinematic viscosity coefficient increases with the temperature, while the right-hand side of (1) decreases with it. The viscosity and density of stars of type B and A hardly differ from those of the Sun but their temperature is essentially higher. According to the above statement there must be instability in these stars. The spectra of many stars belonging to these types show the presence of a strong magnetic field, the reason of which may be found in the instability [2].

It is another question, of course, how the turbulence will be influenced by a disturbing magnetic field (to which it shows instability). It would be difficult to answer this considering that we do not possess adequate experimental bases for it. Similarly, in accordance with the turbulence theory we may describe the motion there, so that the lines of force would be dissolved as the streamlines, that is to say, they would transform themselves into strongly winding curves constantly altering in form. At a certain point the strength and direction of the magnetic field fluctuates constantly.

As shown by *Batchelor*, under certain acceptable neglections, there is an analogy, between vorticity and magnetic field. The equations governing the magnetic field-vector are identical in form with the equations governing the vorticity in an incompressible fluid in the absence of an electromagnetic field. We may derive the fluctuations of the magnetic field from the oscillations of the vorticity by this analogy.

We may refer to the oscillation of the vorticity from the mixing length hypothesis, although it has no physical reality, it can be employed to develop the turbulent viscosity in homogeneous isotropic turbulence.

Hence :

$$\mathrm{rot}\ \mathfrak{v}' = -(\mathfrak{l},\ \mathrm{grad})\ \mathrm{rot}\ \bar{\mathfrak{v}} \tag{2}$$

where $\mathfrak{v}$ is the turbulent velocity, $\bar{\mathfrak{v}}$ is the mean value of velocity, and $\mathfrak{l}$ the mixing length. If we, accept this hypothesis as a rough approximation of the turbulent velocity, we get for the fluctuation of the magnetic field, by the analogy between vorticity and magnetic field, the formula:

$$\mathfrak{H}' = -(\mathfrak{l},\ \mathrm{grad})\ \bar{\mathfrak{H}} \tag{3}$$

where $\mathfrak{H}'$ is the fluctuation of the magnetic field and $\bar{\mathfrak{H}}$ the mean value.

A more exact elaboration of this problem could be reached by introducing the spectral funtion [3]. The present paper should be considered as a preliminary one dealing with the effects observed in turbulent ionized gases. For a further elaboration we have to introduce the spectral function of the fluctuating magnetic field, and the correlations. In the following we shall start from the hydrodynamical equations and from the equations of the electro-magnetic field, and develop the equations for the magneto-hydrodynamics and for the disturbances of the magnetic field.

Fundamental equations

Let us begin with the equation of hydrodynamics completed with Lorentz's law of force and with Maxwell's equations for the electromagnetic field :

$$\frac{d\mathfrak{v}}{dt} = \mathrm{grad}\ V - \frac{\varrho_e}{\varrho}\,\mathfrak{E} - \frac{\mu}{c\varrho}\,[\mathfrak{i},\ \mathfrak{H}] - \frac{1}{\varrho}\,\mathrm{grad}\ p + \nu\Delta\mathfrak{v} \tag{4}$$

$$\frac{d\varrho}{dt} + \varrho\ \mathrm{div}\ \mathfrak{v} = 0 \tag{5}$$

$$c\ \mathrm{rot}\ \mathfrak{H} = \varepsilon\,\frac{\partial\mathfrak{E}}{\partial t} + \mathfrak{i} \tag{6}$$

$$c \operatorname{rot} \mathfrak{E} = -\mu \frac{\partial \mathfrak{H}}{\partial t} \tag{7}$$

$$\varepsilon \operatorname{div} \mathfrak{E} = \varrho_e \tag{8}$$

$$\operatorname{div} \mathfrak{H} = 0 \tag{9}$$

where $\mathfrak{v}$ is the velocity, V the gravitational potential, p the pressure, ϱ the density, $\mathfrak{i}$ the intensity of the electric currents, $\mathfrak{E}$ the electric and $\mathfrak{H}$ the magnetic strength, ε the dielectric constant, μ the magnetic permeability (assuming that ε and μ have the same values at all points), ϱ_e the charge density. Let us complete the above equations by adding Ohm's law for a conductive medium moving in a magnetic field:

$$\mathfrak{i} = \sigma \mathfrak{E} + \frac{\sigma}{c} [\mathfrak{v}, \mathfrak{H}] , \tag{10}$$

where σ is the specific conductivity.

Equations of magneto-hydrodynamics.

a) Laminar motions. From the equations (4)—(10) we may deduce the equations of the conservation of all momenta which may be understood as a more general form of the hydrodynamical equation of motion.

If ε and μ are constants we may write Lorentz's law of force with the aid of Maxwell's tensions in the form:

$$\varrho_e \mathfrak{E} + \frac{\mu}{c} [\mathfrak{i}, \mathfrak{H}] = -\operatorname{grad} \frac{1}{2}(\varepsilon \mathfrak{E}^2 + \mu \mathfrak{H}^2) + \\ + \operatorname{Div} (\varepsilon \{\mathfrak{E}, \mathfrak{E}\} + \mu \{\mathfrak{H}, \mathfrak{H}\}) - \frac{\mu}{c} \frac{\partial [\mathfrak{E}, \mathfrak{H}]}{\partial t} \tag{11}$$

where Div means tensor divergence and $\{\mathfrak{E}, \mathfrak{E}\}$, $\{\mathfrak{H}, \mathfrak{H}\}$ denote the following tensors[1]:

$$\{\mathfrak{E}, \mathfrak{E}\}_{ik} = \mathfrak{E}_i \mathfrak{E}_k , \quad \{\mathfrak{H}, \mathfrak{H}\}_{ik} = \mathfrak{H}_i \mathfrak{H}_k .$$

Let us insert (11) in (4):

$$\frac{\partial}{\partial t}\left(\varrho \mathfrak{v} + \frac{\varepsilon \mu}{c} [\mathfrak{E}, \mathfrak{H}]\right) + \operatorname{Div} (\{\varrho \mathfrak{v}, \mathfrak{v}\} - \varepsilon \{\mathfrak{E}, \mathfrak{E}\} - \mu \{\mathfrak{H}, \mathfrak{H}\}) = \\ = \varrho \operatorname{grad} V - \operatorname{grad}\left(p + \frac{1}{2} (\varepsilon \mathfrak{E}^2 + \mu \mathfrak{H}^2)\right) + \nu \varrho \Delta \mathfrak{v}$$

where

$$\{\varrho \mathfrak{v}, \mathfrak{v}\}_{ik} = \varrho \mathfrak{v}_i \mathfrak{v}_k .$$

[1] More generally the tensorial product of $\mathfrak{A}$ and $\mathfrak{B}$ may be written;

$$\mathsf{T} = \{\mathfrak{A}, \mathfrak{B}\} \quad \text{or} \quad \mathsf{T}_{ik} = \mathfrak{A}_i \mathfrak{B}_k .$$

The so- called Poynting vector on the left-hand side $\frac{1}{c}[\mathfrak{E}, \mathfrak{H}]$ gives the momentum of the electromagnetic field, which, compared with the mechanical momentum, may generally be neglected (if the magnetic and electric fields are not too strong). In the second member of the right-hand side $\varepsilon\,\mathfrak{E}^2 + \mu\,\mathfrak{H}^2$ is the energy of the electromagnetic field which, being a correction of pressure, becomes more important in so far as the electromagnetic field fluctuates.

From Maxwell's equations and Ohm's law a very important equation concerning the magnetic field can be derived. Let us insert (10) into (6) and take the rotation (curl) of both sides :

$$-c\,\Delta\mathfrak{H} = \varepsilon\,\frac{\partial}{\partial t}(\text{rot}\;\mathfrak{E}) + \sigma\;\text{rot}\;\mathfrak{E} + \frac{\sigma\,\mu}{c}\;\text{rot}\,[\mathfrak{v}, \mathfrak{H}].$$

Making use of (7) :

$$c\,\Delta\mathfrak{H} = +\frac{\varepsilon\,\mu}{c}\,\frac{\partial^2\mathfrak{H}}{\partial t^2} + \frac{\sigma\mu}{c}\,\frac{\partial\mathfrak{H}}{\partial t} - \frac{\sigma\,\mu}{c}\;\text{rot}\;[\mathfrak{v}, \mathfrak{H}].$$

If the magnetic field has no high frequency oscillations, $\partial^2\mathfrak{H}/\partial t^2$ may be neglected and we obtain:

$$\frac{\partial\mathfrak{H}}{\partial t} - \text{rot}\,[\mathfrak{v}, \mathfrak{H}] = \frac{c^2}{\sigma\,\mu}\;\Delta\mathfrak{H} \qquad (12)$$

or

$$\frac{\partial\mathfrak{H}}{\partial t} + \text{Div}\,\{\mathfrak{H}, \mathfrak{v}\} - \text{Div}\,\{\mathfrak{v}, \mathfrak{H}\} = \varkappa\,\Delta\mathfrak{H} \qquad (12a)$$

where $\varkappa = c^2/\sigma\mu$.

Batchelor has shown that this equation is analogous to the equation governing the vorticity[4].

We may neglect the terms containig the electric field-vector in equation (10) and so we obtain as the *two fundamental equations of the magneto-hydrodynamics :*

$$\frac{\partial\varrho\mathfrak{v}}{\partial t} + \text{Div}\,\{\varrho\,\mathfrak{v}, \mathfrak{v}\} - \text{Div}\,\{\mu\,\mathfrak{H}, \mathfrak{H}\} = \varrho\;\text{grad}\;V - \text{grad}\;P + \nu\,\varrho\,\Delta\mathfrak{v} \qquad (13)$$

$$\frac{\partial\mathfrak{H}}{\partial t} + \text{Div}\,\{\mathfrak{H}, \mathfrak{v}\} - \text{Div}\,\{\mathfrak{v}, \mathfrak{H}\} = \varkappa\,\Delta\mathfrak{H} \qquad (14)$$

where

$$P = p + \frac{1}{2}(\varepsilon\,\mathfrak{E}^2 + \mu\mathfrak{H}^2).$$

We may write both equations in a more symmetrical form, by introducing new functions. Let us multiply (13) by $1/\varrho$ and (14) by $(\mu/\varrho)^{1/2}$ and add them together

$$\frac{\partial}{\partial t}\left(\mathfrak{v}+\left(\frac{\mu}{\varrho}\right)^{1/3}\mathfrak{H}\right)+$$

$$+\mathrm{Div}\left(\{\mathfrak{v},\mathfrak{v}\}-\frac{\mu}{\varrho}\{\mathfrak{H},\mathfrak{H}\}+\left(\frac{\mu}{\varrho}\right)^{1/2}\{\mathfrak{H},\mathfrak{v}\}-\left(\frac{\mu}{\varrho}\right)^{1/2}\{\mathfrak{v},\mathfrak{H}\}\right)=$$

$$=\mathrm{grad}\ V-\frac{1}{\varrho}\ \mathrm{grad}\ P+\Delta\left(\nu\,\mathfrak{v}+\varkappa\left(\frac{\mu}{\varrho}\right)^{1/2}\mathfrak{H}\right).$$

If ϱ is constant we have:

$$\frac{\partial\mathfrak{T}}{\partial t}+\mathrm{Div}\{\mathfrak{T},\mathfrak{T}^*\}=\mathrm{grad}\ V-\frac{1}{\varrho}\ \mathrm{grad}\ P+\Delta\left(\nu\,\mathfrak{v}+\varkappa\left(\frac{\mu}{\varrho}\right)^{2}\mathfrak{H}\right) \qquad (15)$$

where

$$\mathfrak{T}=\mathfrak{v}+\left(\frac{\mu}{\varrho}\right)^{1/2}\mathfrak{H} \qquad (16)$$

$$\mathfrak{T}^*=\mathfrak{v}-\left(\frac{\mu}{\varrho}\right)^{1/2}\mathfrak{H}. \qquad (17)$$

We obtain a similar equation by subtracting (14) from (13), only $\mathfrak{T}$ and $\mathfrak{T}^*$ must be interchanged:

$$\frac{\partial\mathfrak{T}^*}{\partial t}+\mathrm{Div}\{\mathfrak{T}^*,\mathfrak{T}\}=\mathrm{grad}\ V-\frac{1}{\varrho}\ \mathrm{grad}\ P+\Delta\left(\nu\,\mathfrak{v}-\varkappa\left(\frac{\mu}{\varrho}\right)^{1/2}\mathfrak{H}\right). \qquad (18)$$

We may also derive this equation from (17) by interchanging $\mathfrak{T}$ and $\mathfrak{T}^*$.

According to *Batchelor* in the case of indifferent equilibrium of the hydrodynamical and electrodynamical procedures ν is equal to $\varkappa$, and then we may write for (17) and (18):

$$\begin{aligned}\frac{\partial\mathfrak{T}}{\partial t}+\mathrm{Div}\{\mathfrak{T},\mathfrak{T}^*\}&=\mathrm{grad}\ V-\frac{1}{\varrho}\,\mathrm{grad}\,P+\nu\,\Delta\mathfrak{T}\\ \frac{\partial\mathfrak{T}^*}{}+\mathrm{Div}\{\mathfrak{T}^*,\mathfrak{T}\}&=\mathrm{grad}\ V-\qquad\mathrm{grad}\,P+\nu\,\Delta\mathfrak{T}^*.\end{aligned} \qquad (19)$$

The solutions of these equations seem to be easier than those of (13) and (14). But the solutions of (19) according to the equation of hydrodynamical continuity (9) have to fulfill certain conditions which may be written in the following form:

$$\begin{aligned}\mathrm{div}\,\mathfrak{T}&=0\\ \mathrm{div}\,\mathfrak{T}^*&=0.\end{aligned} \qquad (20)$$

b) Turbulent motion. Based on the hypothesis mentioned in the introduction we may easily pass from the equations of the laminar motion, over to those of the turbulent state. Let us assume that besides the oscillation of velocity, of pressure, etc. the turbulence would be characterized by the fluctuation of the magnetic field. However, in the absence of an external magnetic field the mean value does not necessarily differ from zero. But if the disturbing fields are slightly adjusted (having approximately the same directions) the mean value will not equal zero. In the following we assume that the mean value of the fluctuating field is not zero.

Let us take $\overline{\mathfrak{v}}$ and $\overline{\mathfrak{H}}$ for the mean values of the velocity and of the magnetic field-vector, their turbulent perturbations should be denoted by $\mathfrak{v}'$ and $\mathfrak{H}'$ we may write:

$$\mathfrak{v} = \overline{\mathfrak{v}} + \mathfrak{v}' \qquad \mathfrak{H} = \overline{\mathfrak{H}} + \mathfrak{H}'.$$

Insertig these expression into (12)

$$\frac{\partial \overline{\mathfrak{H}}}{\partial t} + \frac{\partial \mathfrak{H}'}{\partial t} - \mathrm{rot}\,[\overline{\mathfrak{v}}, \overline{\mathfrak{H}}] - \mathrm{rot}\,[\overline{\mathfrak{v}}, \mathfrak{H}'] - \mathrm{rot}\,[\mathfrak{v}', \overline{\mathfrak{H}}] - \mathrm{rot}\,[\mathfrak{v}', \mathfrak{H}'] = \\ = \varkappa\, \Delta \overline{\mathfrak{H}} + \varkappa\, \Delta \mathfrak{H}'.$$

We take mean values (supposing that $\overline{\mathfrak{v}'} = 0$ and $\overline{\mathfrak{H}'} = 0$):

$$\frac{\partial \overline{\mathfrak{H}'}}{\partial t} - \mathrm{rot}\,[\overline{\mathfrak{v}}, \overline{\mathfrak{H}}] = \varkappa\, \Delta \overline{\mathfrak{H}} + \mathrm{rot}\,\overline{[\mathfrak{v}', \mathfrak{H}']}. \tag{21}$$

Inserting (3) into the last member of the right-hand side:

$$[\mathfrak{v}', \mathfrak{H}'] = -[\mathfrak{v}', (\mathfrak{l}, \mathrm{grad})\,\overline{\mathfrak{H}}] = \Lambda_{lk} \frac{\partial \mathfrak{H}_i}{\partial x_k} - \Lambda_{ik} \frac{\partial \mathfrak{H}_l}{\partial x_k}$$

where Λ_{ik} denotes the following tensor:

$$\Lambda_{ik} = \overline{\mathfrak{v}_i\, \mathfrak{l}_k} \tag{22}$$

which formally corresponds to the exchange-tensor of the turbulent motion. Assuming again that, in the case of isotropic turbulence only the diagonal terms of Λ_{ik} are not equal to zero, that is

$$\Lambda_{ik} = \begin{cases} \Lambda, & i = k \\ 0, & i \neq k \end{cases} \qquad \Lambda = \overline{|\mathfrak{v}'|\,|\mathfrak{l}|}$$

then the equation becomes essentially simplified:

$$[\mathfrak{v}', \mathfrak{H}'] = -\Lambda\, \mathrm{rot}\, \overline{\mathfrak{H}}.$$

Inserting it into (21) if Λ is constant:

$$\frac{\partial \mathfrak{H}}{\partial t} - \mathrm{rot}\,[\mathfrak{v}, \mathfrak{H}] = (\varkappa + \Lambda)\, \Delta \mathfrak{H}.$$

The role of Λ is, therefore, analogous to that of $\varkappa$. Its expression resembles that of the turbulent viscosity coefficient. If we assume that l is identical with the mixing length, then Λ will be identical with the coefficient of the turbulent viscosity. In this case besides Λ we may also neglect $\varkappa$ which is of the same order of magnitude as the molecular viscosity. The equation (22) would correspond to the condition of equilibrium in (1). The mean distance of velocity-disturbances (eddies) would correspond to that of the disturbances of the magnetic field (dipoles) ; this may be understood as the coincidence of the eddies and of the disturbing magnetic field. As mentioned in the introduction the development of such a system could only be explained if the temperature in the centre of an eddy were lower than that of its neighbourhood.

The turbulent conductivity and magnetic permeability

As I have mentioned, the mean value of the magnetic-disturbances will differ from zero, if they tend to arrange themselves. We may accept criterion (1) for the increase of this field. In this so called macroscopic field the macroscopic motion of the medium would correspond to the molecular phenomena. The turbulent viscosity corresponds to the molecular viscosity, the damping of the magnetic energy due to the turbulence corresponds to the Joule heat loss. We have expressed the diffusion of the magnetic field due to the molecular phenomena by means of the specific conductivity and the magnetic permeability. The question arises here as to whether the macroscopic damping in turbulence could not be solved by introducing similar quantities. Such supposed quantities would be the turbulent conductivity and the turbulent magnetic permeability.

After this we may write Λ in the following form:

$$\Lambda = \frac{c^2}{MS} \tag{23}$$

where M is the turbulent magnetic permeability and S the turbulent conductivity. If the former theory is correct, then Λ will be equal to the turbulent viscosity coefficient. The value of which, as is well known, may also be about 10^6 times higher than the corresponding molecular quantity. Consequently the product MS has to differ from $\mu\sigma$ by the same order. Based upon certain electromagnetic peculiarities of the turbulent elements I have referred earlier to such a change. (Supposing the turbulent elements to possess charges as well as magnetic momenta, further charge transports can be possible only by means of macroscopic motion of the turbulent elements. The electrons only diffuse towards the turbulent elements and their motions towards the external field may be neglected. But I did not succeed in an exact elaboration.

In the meantime appeared *Sweet's* paper [5] dealing with a similar problem. It was shown by him that conductivity is diminished by several orders of magnitude due to the rapid fluctuation of the turbulent motion and magnetic field. According to his approximation the specific conductivity in the convective layer of the Sun is diminished by at least 10^6 as a result of turbulence. Therefore, if conductivity was originally 10^{12} it would diminish to 10^6 in turbulence.

This value, although only a rough estimate, suited my investigations. If we take the electric charges of the sunspots equal to 10 coulombs, as necessary to build up their magnetic fields, we obtain a change of the same order of magnitude in accordance with the former inexact theory.

If conductivity based on (23) is known we may calculate the changed value of the magnetic permeability as well. According to (23):

$$M\,S = \frac{c^2}{A/\varrho} = 10^8$$

(A/ϱ being the turbulent viscosity coefficient) from which taking for the value of the conductivity 10^6 we get:

$$M = 1 + 10^2 \sim 10^2.$$

This result, though it still needs proving, appears to be remarkable from two points of view. Permeability being higher than the unit, it is independent of the motion of the medium (ions). Namely if the magnetic field is built up by the motion of the ions simultaneously with the magnetic momenta then momenta of momentum would occur, lending to the field thereby a diamagnetic peculiarity [6]. On the other hand, only ferromagnetic media possess such a high permeability as is well known, where in the case of specially arranged elementary magnetic dipoles, a permanent external field built up. Such an arrangement is not at all trivial in turbulence, but the mean value of the disturbing field would differ from zero only then when the fields within a suitable large volume element do not destroy each other. In the following we have assumed the presence of such a special arrangement, but the correctness of this assumption has not been proved. Perhaps we may hope to prove it by means of a more exact elaboration of the turbulence theory. For the moment the correctness of this hypothesis could most probably be examined by means of experiments. Unfortunately it would be very difficult to compile such an experiment because the turbulence of gases of very high temperature must be examined. These highly ionized gases show an excessive tendency to chemical reaction in consequence of their high temperature (some thousand degrees) and could therefore attack the substance of the equipment producing the stream.

Thus in the following we have to accept the arrangement of the magnetic disturbances as the most important hypothesis of the present theory.

Accepting this hyopthesis it would de conceivable at once that the magnetic field of the stars results from high ionization and turbulence. In any case this inference is not inconsistent with the observations as the strongest magnetic fields have been measured in stars of types B and A which showed also the highest Doppler shift resulting from the turbulence. The magnetic field of the Sun and its turbulent state compared to those of the above stars is small in accordance with the hypothesis. The magnetic field of the Earth may also be similar. We may infer the turbulence from the very high viscosity of the interior of the Earth[7].

Based on this hypothesis, we may also draw conclusions directly, as to the above high value of magnetic permeability. The energy of an external field, as is well known, will be changed by the permeability of the medium, if we take $\mathfrak{H}^2$ for the energy per unit volume in vacuo, then in a medium with permeability μ it will change to $\mu\,\mathfrak{H}^2$. Therefore the energy of the field will increase, if the permeability is higher than the unit, and it will decrease if lower. Similarly permeability may also be understood in the turbulence. If the energy of the magnetic field increases after turbulence has come into operation, the medium will show paramagnetic pecularities, and diamagnetic ones if the energy decreases.

Let us assume that the permeability of a gas in absence of turbulence is equal to the unit. The strength of the field in turbulence would be constantly fluctuating in consequence of the magnetic disturbances. Therefore we may write:

$$\mathfrak{H} = \mathfrak{H}_o + \mathfrak{H}'$$

where $\mathfrak{H}_o$ is the mean value composed of the external field and the mean strength of disturbances. The energy of the field will be:

$$\mathfrak{H}^2 = \mathfrak{H}_o^2 + 2\,\mathfrak{H}_o\,\mathfrak{H}' + \mathfrak{H}'^2.$$

Let us assume that $\mathfrak{H}' = 0$. If we consider $\mathfrak{H}$ as a vector of magnetic induction we may write:

$$\mathfrak{H} = M\,\mathfrak{H}_o$$

from which:

$$M = 1 + \left(\frac{\mathfrak{H}'}{\mathfrak{H}_o}\right)^2.$$

The mean value of the disturbances being essentially lower than the external field, $\mathfrak{H}^2$ contains only the energy of the external field. It follows from the above formula that M will always be larger than the unit, proving that the turbulent state becomes paramagnetic.

Considering the phenomena of the solar surface it seems to be probable that sunspots produce the magnetic field of the Sun for an external field so:

$$\mathfrak{H}'/\mathfrak{H}_2 \sim 10$$

from which

$$M = 1 + 10^2 \sim 10^2$$

which agrees with the foregoing calculations referring to order of magnitude.

Magneto-hydrodynamic equations in turbulence

We have examined above the quantitative changes of the conductivity and permeability in turbulence. Let us develop in the following, and change the fundamental equation (19) by using the hypothesis (2) and (3). In order to amplify the calculations we take instead (2) the formula

$$\mathfrak{v}' = -(\mathfrak{l}, \mathrm{grad})\, \bar{\mathfrak{v}}, \tag{25}$$

which in case of isotropic homogeneous turbulence would also be correct.

The equations concerning the mean values will be :

$$\begin{aligned} \frac{\partial \mathfrak{T}}{\partial t} - \mathrm{Div}\{\mathfrak{T}, \mathfrak{T}^*\} &= \mathrm{grad}\, V = \frac{1}{\varrho}\, \mathrm{grad}\, P - \mathrm{Div}\{\mathfrak{T}, \mathfrak{T}^{*\prime}\} \\ \frac{\partial \mathfrak{T}^*}{\partial t} + \mathrm{Div}\{\mathfrak{T}^*, \mathfrak{T}\} &= \mathrm{grad}\, V - \frac{1}{\varrho}\, \mathrm{grad}\, P - \mathrm{Div}\{\mathfrak{T}^*, \mathfrak{T}\} \end{aligned} \tag{26}$$

(taking the molecular viscosity zero and conductivity infinite). Making use of the equation (3) and (25) standing for $\mathfrak{T}$ and $\mathfrak{T}^*$:

$$\mathfrak{T} = -(\mathfrak{l}, \mathrm{grad})\, \bar{\mathfrak{T}}$$

$$\mathfrak{T}^* = -(\mathfrak{l}, \mathrm{grad})\, \bar{\mathfrak{T}}^*.$$

Utilising these formulae $\{\mathfrak{T}', \mathfrak{T}^{*\prime}\}$ may be transformed as follows[1]:

$$\{\mathfrak{T}', \mathfrak{T}^{*\prime}\} = -\{(\mathfrak{l}, \mathrm{grad})\, \bar{\mathfrak{T}}, \mathfrak{T}^{*\prime}\} = -\{\mathrm{grad}, \bar{\mathfrak{T}}\} \cdot \{\mathfrak{T}^{*\prime}, \mathfrak{l}\}.$$

Or denoting it in a tensorial form

$$\{\mathfrak{T}^{*\prime}, \mathfrak{l}\}_{ik} = \mathfrak{T}_i \mathfrak{l}_k \qquad \text{and} \qquad \{\mathrm{grad}, \bar{\mathfrak{T}}\}_{ik} = \frac{\partial \bar{\mathfrak{T}}_k}{\partial x_i}$$

so

$$\mathrm{Div}\{\mathfrak{T}', \mathfrak{T}^{*\prime}\} = \frac{\partial \{\mathfrak{T}', \mathfrak{T}^{*\prime}\}_{ik}}{\partial x_k} = -\frac{\partial \mathfrak{T}^{*\prime} \mathfrak{l}_j}{\partial x_k} \cdot \frac{\partial \mathfrak{T}_i}{\partial x_j} - \mathfrak{T}_k^{*\prime} \mathfrak{l}_j \frac{\partial^2 \mathfrak{T}_i}{\partial x_j \partial x_k}.$$

Inserting it into (26) we obtain the equation for magneto-hydrodynamics in turbulent motion :

$$\frac{\partial \mathfrak{T}}{\partial t} + \mathrm{Div}\{\mathfrak{T}, \mathfrak{T}^*\} = \mathrm{grad}\, V - \frac{1}{\varrho}\, \mathrm{grad}\, P + A^* \Delta \mathfrak{T}. \tag{28}$$

In the case of homogenous isoptropic turbulence only the diagonal elements of the tensor $\{\mathfrak{T}^{*\prime}, \mathfrak{l}\}$ are not zero and these will also be constant, that is

$$\mathfrak{T}_i^{*\prime} \mathfrak{l}_k = \begin{cases} A^* & i = k \\ 0 & i \neq k \end{cases}$$

and so

$$\mathrm{Div}\{\mathfrak{T}', \mathfrak{T}^{*\prime}\} = -A^* \Delta \mathfrak{T}. \tag{27}$$

[1] The dot here signifies the tensorial (transformation) product of two tensors.

$$\frac{\partial \mathfrak{T}^*}{\partial t} + \mathrm{Div}\{\mathfrak{T}^*, \mathfrak{T}\} = \mathrm{grad}\, V - \frac{1}{\varrho}\,\mathrm{grad}\, P + A\,\Delta\mathfrak{T}^* \tag{29}$$

where

$$A = \overline{|\mathfrak{T}'|\,|\mathfrak{l}|} = \overline{|\mathfrak{v}'|\,|\mathfrak{l}|} + \left(\frac{\mu}{\varrho}\right)^{1/2} \overline{|\mathfrak{H}'|\,|\mathfrak{l}|}$$

$$A^* = \overline{|\mathfrak{T}^{*\prime}|\,|\mathfrak{l}|} = \overline{|\mathfrak{v}'|\,|\mathfrak{l}|} - \left(\frac{\mu}{\varrho}\right)^{1/2} \overline{|\mathfrak{H}'|\,|\mathfrak{l}|}.$$

The equations under (20) remain unaltered.

The solution of equations (28) and (29) for a rotating star seem to be very difficult. We have to calculate the gravitational potential from Poisson's equation and the pressure from the equation of state. Hereafter there are still six unknowns, the components of $\mathfrak{T}$ and $\mathfrak{T}^*$. We may eliminate three of them by transforming the system into a system of fourth order. One of the systems of the unknowns is contained linearly in (28) the other in (29). We may calculate them simply algebraically and inserting the results into the other system of equations, we obtain a system of differential equations of the fourth order.

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МАГНИТНЫЙ ЭФФЕКТ ТУРБУЛЕНТНОГО СОСТОЯНИЯ В ИОНИЗИРОВАННЫХ ГАЗАХ

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Резюме

В газах с хорошей электропроводностью (ионизированных) вследствие турбулентного состояния возникают мгновенные локальные магнитные поля. Эти поля вызываются диффузией электронов, возникающей вследствие колебания температуры. Из критерия стабильности Бечелора можно определить, что легкоионизирующиеся газы при температуре около 10.000° Ц являются нестабильными по отношению к магнитным помехам. На основе аналогии интенсивности турбуленции и напряженности магнитного поля с помощью теории вихревого транспорта можем оценить те эффекты, которые возникают в газе, обладающем хорошими проводящими свойствами. Можем сделать вывод, что удельная проводимость газа должна падать с некоторыми порядками в то время когда магнитная проницаемость должна возрастать. Теория дает необходимое условие того, что магнитное поле звезд происходит от ионизации и от турбулентного состояния.

Уравнения электромагнитной гидродинамики с введением новых функций могут быть записаны в симметричной форме, что является аналогичным уравнениям движения классической гидродинамики. Уравнения остаются аналогичными в случае гомогенной изотопной турбуленции.