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ON THE ORIGIN OF THE MAGNETIC FIELD OF THE SUNSPOTS

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A NAPFOLTOK MÁGNESES TERÉNEK EREDETE

(Összefoglalás)

A megfigyelés azt mutatja, hogy az igen kicsi napfoltok folytonos átmenetet képeznek a sötét granulákkal. Ebből arra következtethetünk, hogy a napfoltok a sötét granulákból alakulnak ki oly módon, hogy azok valamilyen ok miatt hirtelen növekedni kezdenek. Egy ilyen ok lehet pl. egy hirtelen fellépő hidrodinamikai örvény, amelyik a sötét granulában lép fel és kinetikus energiáját környezetének termikus energiájából meríti. A hőmérséklet csökkenése miatt elektrondiffúzió indul meg a környezetből az örvény felé és így ebben tértöltés keletkezik. Az örvénymozgás miatt fellépő mágneses tér a hőmérsékletet tovább csökkenti úgy, hogy a kezdeti kicsi hidrodinamikai örvény hamarosan napfolttá alakul.

A kvalitatív vizsgálat azt mutatja, hogy a stacionárius állapotban, midőn a diffúziós nyomás egyensúlyban van a tértöltés elektrosztatikus taszításával, a napfoltok centrális mágneses terének arányosnak kell lennie a napfolt sugarával. Az elméletnek ez az eredménye igen jól egyezik a tapasztalattal. A stacionárius állapot a napfoltok méretére felső határt ad, mely azonban nagyobb, mint a napfoltok átlagos mérete. Ebből arra lehet következtetnünk, hogy a napfoltok energiája fejlődésük közben szétszóródik, még mielőtt a stacionárius állapotot felvehetné.

ON THE ORIGIN OF THE MAGNETIC FIELD OF THE SUNSPOTS

Introduction. It is known that the smallest sunspots represent a transition to the dark granulae. This fact leads us to the assumption that sunspots originate from the granular structure in such a manner that the dark intergranular field increases rapidly in consequence of some reason. Such a reason may be given, for example, by suddenly produced hydrodynamic eddy obtaining its kinetic energy from the thermal energy of the photosphere. But the most important disadvantage of such a theory lies in the fact that it renders the explaining of the origin of the magnetic field impossible. On the other hand, the magnetic field of the larger sunspots needs subsist both before and after the visibility of the spots. Therefore the field of the large sunspots cannot develop from the granular structure in the course of a few hours or days. It must have propagated as a magnetic disturbance existing long before and producing similar disturbances in the surrounding granulae. The individual smaller sunspots, of course, can develop directly from the granulae.

It was *Alfvén* who demonstrated for the first time that condition of a magneto-hydrodynamical equilibrium was to keep the sum of the pressure and that of the magnetic energy constant. Therefore, in equilibrium, the magnetic field may increase only in case when pressure diminishes. This means (if density is constant) that it may increase only at the expense of the thermal energy. Consequently the growth of the dark intergranular field demonstrates the presence of a magnetic field.

In the following a new theory of the magnetic field of the sunspots will be proposed, supposing that a sunspot which grows out of a dark granula, is a simple hydrodynamical whirl. The temperature in the whirl is lower than in its surroundings. This fall of temperature will be sufficient to generate spontaneously a magnetic field which in its turn will further reduce the temperature.

Mechanical and ionization equilibrium in sunspots. The photosphere and the sunspots are composed of a mixture of electrons and of neutral as well as of ionized gases. Ionization is virtually of thermal origin and consequently its degree depends upon temperature, density being constant. The degree of ionization in the interior of sunspots is smaller than in the surrounding photosphere. If we consider the conditions of ionization equilibrium only, the distribution of the density of electrons and that of ions will be stable. However, the hydrodynamical equations hold separately for each constituent. In ionized gases the electrons follow the Boltzmann statistics and may be considered as one of the constituents of the mixture. But in this case, if we neglect gravitational and electromagnetic influences, electrons and ions intend to occupy the whole field with uniform density (pressure being constant everywhere), consequently a diffusion of electrons and ions tow rds the sunspot will take place to eliminate the differences of density. But as the velocity of electron diffusion is several thousand times higher than that of the diffusion of ions, during the first phase of the diffusion there will be a prevalance of electrons in the sunspots, that is, if we assume that the electrons too parti ipate in the hydrodynamical eddy-motion of sunspots, magnetic fields are developing too. As it has been already mentioned above, the presence of a magnetic field leads to further diminution of temperature, however, increases the diffusion and intensifies the magnetic field which leads to a further reduction of temperature. An originally simple hydrodynamical eddy will be transformed very rapidly into a sunspot.

However, the size of a sunspot increases rapidly only in the beginning. The electrostatic repulsions, occuring in the later phase bring the diffusion to a complete stop. Thus there will be an approximately steady state of equilibrium in which the presence of diffusion of will be equal to electromagnetic repulsion. The diffusion ions, however, reduces the charge. This results in the diminution of the magnetic strength too. Let us now examine a simplified case when the latter phenomenon is not taken into consideration.

Fundamental equation. Let us examine a vertical eddy in the photosphere. The temperature in the interior of the eddy will be lower than in the surrounding. Let us suppose that the gas is ionized, i. e., it contains free electrons which follow the Boltzmann statistics.

Therefore the gas will be a mixture composed of electrons, neutral atoms and ions. For each constituent of this mixture holds the equation of motion :

$$n_i m_i (\mathfrak{v}, \text{grad})\, \mathfrak{v}_i = n_i m_i \,\text{grad}\, V - \text{grad}\, p_i + Z_i\, e\, n_i\, \mathfrak{E} + \frac{1}{c}\, [\mathfrak{i}_i, \mathfrak{H}] \qquad (1)$$

where $\mathfrak{v}$ is the velocity of the mixture, $\mathfrak{v}_i$ the mass-velocity of the i-th constituent, n_i the number of the particles (in unite volume), m_i the mass of the individual particles. eZ_i is the charge of the individual particles (e is the elementary charge and $Z = -1$ for electrons, $Z = 1, 2, 3, \ldots$ for ions, $Z = 0$ for neutral particles). p_i is the partial pressure, $\mathfrak{E}$ and $\mathfrak{H}$ the total electric and magnetic field strengths, $\mathfrak{i}$ the intensity of the electric current composed of the charge-displacement produced by diffusion and the induced electric currents. Let us neglect the latter, $\mathfrak{i}$ is to be expressed by the velocity of the individual components of the mixture :

$$\mathfrak{i} = \Sigma\, \mathfrak{i}_i = \Sigma\, n_i\, e\, Z_i\, \mathfrak{v}_i\,. \qquad (2)$$

The hydrodynamical equation for the whole gas-mixture can be written :

$$\varrho\, (\mathfrak{v}, \text{grad})\, \mathfrak{v} = \varrho\, \text{grad}\, V - \text{grad}\, p + \varepsilon\, \mathfrak{E} + \frac{1}{c}\, [\mathfrak{i}, \mathfrak{H}] \qquad (3)$$

where ϱ is the density, p the total pressure (the radiation pressure neglected) and ε the free charge in unit volume :

$$\varepsilon = \Sigma\, n_i\, Z_i\, e\,.$$

For $\mathfrak{E}$ and $\mathfrak{H}$ we have the Maxwellian equations :

$$\text{rot } \mathfrak{H} = \frac{\mathfrak{i}}{c} \tag{5}$$

$$\text{rot } \mathfrak{E} = 0 \tag{6}$$

$$\text{div } \mathfrak{E} = \varepsilon \tag{7}$$

$$\text{div } \mathfrak{H} = 0 \, . \tag{8}$$

The accurate solution of these equations meats with great difficulties. But the calculation can be performed relatively easily in the case of a vertical eddy after the following simplifications :

1. The whole system is symmetrical around a vertical axis and all quantities depend only of the distance from the axis.

2. The whole system (electrons, ions and neutral gas-atoms) is circulating around the symmetrical axis with a constant angular velocity.

3. We examine only the stationary state in which the electrostatic repulsion balances the pressure of diffusion.

After these simplifications we have the equations :

$$\frac{dp_e}{dR} + n_e' e E = 0 \tag{9}$$

$$\frac{dH}{dR} = \frac{n_e' e \omega R}{c} \tag{10}$$

$$\frac{1}{R} \frac{d}{dR} (RE) = - n_e' e \, . \tag{11}$$

Here n_e' denotes the number of the inflowing electrons, p_e denotes the electron pressure in ionisation equilibrium, the modification produced by the diffusion being neglected. E and H denote the magnitude of the electric and of the magnetic field strength. Neglecting the electric field, the equation of motion (4) for equilibrium will be :

$$p + \frac{1}{2} \omega^2 R^2 + \frac{1}{2} H^2 = p_o = \text{constant.}$$

If we neglect the second term, we obtain for the hydrodynamic pressure:

$$\text{p} = p_o - \frac{1}{2} H^2$$

where p_0 denots the pressure outside of the sunspot.

The electric field. The electric field arising from the electron diffusion can be determined by making use of equations (9) and (11)

$$2\frac{dp_e}{dR} - \frac{1}{R^2}\frac{d}{dR}(RE)^2 = 0$$

After integration we have

$$E = \frac{\sqrt{2}}{R}\sqrt{R^2 p_e - 2\int_0^R R p_e \, dR}\,. \tag{13}$$

We see from this equation that the electric field is a funct'on of the electron-pressure. However, if we take pressure for constant this field according to (13) will be identically zero. The field may be built up only if the pressure-gradient does not vanish. Such a distribution is represented by the following formula :

$$p_e = p_{eo} - p'_{ec}\, e^{-\left(\frac{R}{R_1}\right)^2} \tag{14}$$

where p_{eo} denotes the electron pressure outside the sunspot and $p'_{ec} = p_{ec} - p_{eo}$ where p_{ec} means the electron pressure in the centre of the sunspot. R_1 can be identified with the radius of the sunspot. Making use of (14) we get for the electric field :

$$E = \frac{\sqrt{2}}{R}\sqrt{R_1^2 p'_{ec} - p'_{ec}(R^2 + R_1^2)\, e^{-\left(\frac{R}{R_1}\right)^2}}\,.$$

The constant of integration is determined in such a way that E along the axis of the sunspot should be equal to zero.

The magnetic field. As we have supposed, the gas in the sunspot is circulating around the axis. As the eddy possesses electric charge, a magnetic field has to appear. In accordance with (10) and (11) we can write :

$$\frac{dH}{dR} = \frac{\omega R}{c} n'_e e = -\frac{\omega}{c}\frac{d}{dR}(RE)\,.$$

After integration supposing the angular velocity is constant we obtain.

$$H = -\frac{\omega}{c} RE + H_c \tag{15}$$

where H_c denotes the magnetic field along the axis. The magnetic field vanishes at the infinity only if

$$H_c = \frac{\omega}{c}(ER)_{R \to \infty} \tag{16}$$

and so we have

$$H = -\frac{\omega\sqrt{2}}{c}\sqrt{R^2 p_e - 2\int_0^R R p_e \, dR} + H_c\,. \tag{17}$$

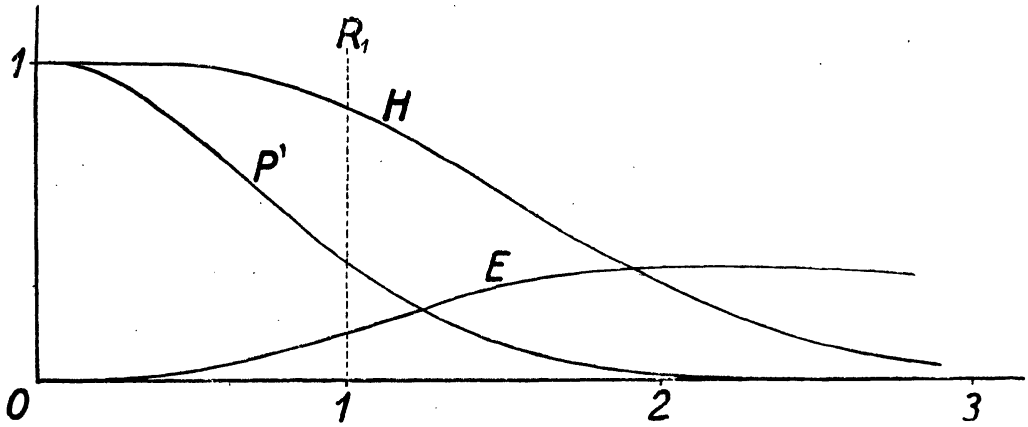
Taking the error-function for p'_e we get for the magnetic field :

$$H = -\frac{\omega\sqrt{2p'_{ec}}}{c}\sqrt{R_1^2 - (R^2 + R_1^2)\, e^{-\left(\frac{R}{R_1}\right)^2}} \tag{18}$$

and in the centre of the sunspot :

$$H_c = \frac{\omega}{c}\sqrt{2p'_{ec}} \cdot R_1 \,. \tag{19}$$

The distribution of H in the sense of (18) is shown in the figure.



Discussion. As shown in (19) the central magnetic field depends on the radius of the sunspots, further on p'_{ec} and on the angular velocity. From all these quantities we are able to observe only the radius of the sunspots.

Making use of the theory of the ionization equilibrium we may estimate the value of p'_{ec}. In ionization equilibrium we have on order of magnitude :

$$p_e \sim b\,T^{5/2}, \qquad b = 7 \cdot 45.10^{-9}$$

hence

$$p'_e = \frac{5}{2}\, b\, T^{3/2}\, T' \tag{20}$$

where T' denotes the difference between the central temperature of the sunspot and the temperature of the surrounding photosphere. However, few observations show that there is no systematic depencende between the size of the sunspots and their central temperature. Therefore the value of T' will be approximately the same for every sunspot. Similarly it is probable that the value of ω should be nearly the same in different sunspots. Accordingly, *the central magnetic field of the sunspots is proportional to their radius.* The existence of such a relationship is in accordance with observation. On table I. we have compiled the values for the central magnetic field of different sunspots[1] and for R_1/H_c :

[1] *Hoyle :* Some Recent Researches in Solar Physics p. 4.

TABLE I.

Radius of the penumbra in cm	Central Magnetic Strength in Gauss	R_1/H_c
3.10^8	$5,0.10^2$	$7,0.10^5$
$5,0.10^8$	$1,0.10^3$	$5,0.10^5$
$8,0.10^8$	$1,5.10^3$	$5,3.10^5$
$1,1.10^8$	$2,0.10^3$	$5,5.10^5$
$1,5.10^8$	$2,5.10^3$	$7,0.10^5$
$2,0.10^8$	$3,0.10^3$	$6,7.10^5$

Therefore R_1/H_c can be considered as constant with the mean value $6,0.10^{+5}$. That means that between R_1 and H_c exists the empirical relationship :

$$H = \frac{1}{6} 10^{-5} R_1 \,. \tag{21}$$

This empirical relation is exactly identical to our theoretical formula (19). It is of great importance, that the derivation of this formula was based on *purely mechanical effects*. The simplifications I have made do not affect the physical basis of the problem.

Making use of (20) and (21) we get :

$$\sqrt{5}\, b\, \omega\, (T_c')^{1/2}\, T_0^{3/4} = 6,0.10^5$$

from which if we take $b = 7,45.10^{-9}$, $T_0 = 5,74.10^3$ and $T' = 1,12.10^3$ we obtain :

$$\omega = 2,57.10^{-6}.$$

This value for the angular velocity is very small. It is very interesting that it is equal to the Sun's equatorial angular velocity. But the present paper is not so exact as to enable us to draw any farther conclusions from it. We may, however, conclude that the velocity must be very low so that it cannot be observed directly or spectroscopically.

Upper limit of the size of sunspots. Above we have only examined the question what kinds of electric and magnetic fields may be produced by a given decrease of the electron pressure. But we have mentioned that the magnetic field diminishes the temperature of the eddy and the electron pressure. Therefore the pressure of diffusion will increase together with the magnetic field and the size of the sunspot.

If we write in (12) for the pressure :

$$p = \frac{\varrho\, \mathfrak{R}}{\mu} T \,.$$

we obtain for the temperature :

$$T' = \frac{\mu}{2\, \varrho\, \mathfrak{R}} H^2. \tag{22}$$

We get from (20) and (21) :

$$H_o^2 = 5\,b\omega^2 R_1^2 .\, T_o^{3/2}\, T'.$$

But (22) and (23) can stand together only if

$$5\,b\,\omega^2 R_1\, T_0^{3/2} = 2\,\frac{\varrho\,\Re}{\mu}$$

hence we obtain for the upper limit of the sunspots :

$$R_1 = 3{,}8.10^{10} \text{ cm.}$$

This value is ten times greater than the radius of the largest sunspots. From this we may conclude that the energy of the developing sunspots disperses before taking up the state of equlibrium wich we have examined above.

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